A Study of Differential Signaling: Stable and Accurate Mixed-Mode Conversion and Extraction of Differential S-Parameters

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***Abstract*—Low voltage differential signaling (LVDS) in high-speed digital systems is utilized to effectively reduce EMI and improve signal quality. Mixed-mode S-parameters are a more general way to characterize a differential network. Therefore, an accurate extraction of mixed-mode S-parameters from single-ended S-parameters is critical for Signal and Power Integrity co-simulation where SSN is generated mainly by high-frequency interconnects. The standard conversion between mixed-mode and single-ended S-parameters involves inversion of a transformation matrix. If there is no coupling, this transformation matrix is orthogonal and numerical inversion can be done accurately. In the presence of coupling, the transformation matrix depends on S-parameters and may become ill-conditioned, i.e. has high condition number, for some values of physical parameters resulting in unstable inversion of the transformation matrix and leading to highly inaccurate converted mixed-mode S-parameters. To analyze the possibility of ill-conditioning, we consider two cases: broadside coupled striplines and coupled microstrip pairs. We find that in both cases when two transmission lines are strongly coupled, the condition number becomes very large. In this case, regularized methods from the theory of ill-posed problems should be used, for example, the truncated SVD method, to obtain accurate mixed-mode S-parameters.**

***Index Terms*—differential S-parameters; S-parameters; signal integrity; conversion matrix; accurate conversion; condition number**

I. Introduction

Differential signaling is the use of two independent transmission lines for one signal, one line carrying one bit and the other one carrying its complement. The measured signal is the difference between the two lines. With the increasing demand for high performance electronic devices, the use of differential signaling is increasing. This trend is largely due to the fact that differential signaling can help meet the performance requirements of these systems by reducing noise. Since mixed mode S-parameters can be an effective way to represent these differential networks, there has been an increasing level of interest in their use.

Mixed-mode scattering parameters (S-parameters) were first introduced by Bockelman and Eisenstadt in 1995 [1]. In 1997 Bockelman continued his work and provided a conversion formula for mixed-mode S-parameters from single-ended S-parameters [2] for the case with no coupling. A lot of research has been done on this topic including theoretical development, conversion from and to single-ended S-parameters, and simulation and measurement comparisons [3], [4], [5], [6], [7]. It is clear from this literature that the coupling between the interconnects plays a critical part in this conversion. If coupling is included, the differential S-parameters deviate significantly from those when coupling is not taken into account. Ideally one would measure or simulate mixed mode S-parameters directly. However, in some cases that is not possible or is impractical; in these cases measurement or simulation of single ended S-parameters followed by conversion to mixed mode S-parameters can be useful.

The conversion from single-ended to mixed mode S-parameters when coupling is present was introduced in [3]. It requires inversion of a transformation matrix that depends in turn on S-parameters. It is known that direct numerical inversion of a matrix may be unstable especially in the case when such matrix is ill-conditioned. If this is not taken into account, converted mixed-mode S-parameters may be highly inaccurate. For example, small errors that inevitably exist in the simulated or measured single ended S-parameters can result in large errors in the mixed mode S-parameters. In such cases, the direct numerical inversion should be avoided and regularized methods from the theory of ill-posed problems [8] should be used. For example, the truncated singular value decomposition (SVD) method that would allow one to regularize the problem, and obtain accurate and reliable results.

The paper is organized as follows. Section II provides a theoretical background on single-ended and mixed-mode S-parameters conversion. A model problem with broadside coupled striplines is used in Section III to analyze the condition number of the transformation matrix. Another example includes coupled microstrip pairs presented in Section IV. Finally, in section V we provide our conclusions.

II. Single-Ended and Mixed-Mode S-Parameters

In this section, we present some fundamental concepts on single-ended and mixed mode S-parameters using a four-port single-ended network shown in Fig. 1, where port 1 and 2 represent the differential input and port 3 and 4 represent the differential output. Port 1 and 3 are positive voltage references, port 2 and 4 are negative ones. The conversion between single-ended S-parameters and mixed-mode S-parameters, denoted

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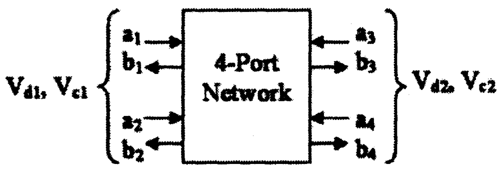

Fig. 1. Four-port network.

by S^{mm}, for this four-port network is provided by [3]:

$$S^{mm} = (M_1 S + M_2)(M_2 S + M_1)^{-1} \tag{1}$$

where

$$M_1 = \begin{pmatrix} \frac{1+k_{oo}}{2\sqrt{2k_{oo}}} & -\frac{1+k_{oo}}{2\sqrt{2k_{oo}}} & 0 & 0 \\ 0 & 0 & \frac{1+k_{oo}}{2\sqrt{2k_{oo}}} & -\frac{1+k_{oo}}{2\sqrt{2k_{oo}}} \\ \frac{1+k_{oe}}{2\sqrt{2k_{oe}}} & \frac{1+k_{oe}}{2\sqrt{2k_{oe}}} & 0 & 0 \\ 0 & 0 & \frac{1+k_{oe}}{2\sqrt{2k_{oe}}} & \frac{1+k_{oe}}{2\sqrt{2k_{oe}}} \end{pmatrix}$$

$$M_2 = \begin{pmatrix} \frac{1-k_{oo}}{2\sqrt{2k_{oo}}} & -\frac{1-k_{oo}}{2\sqrt{2k_{oo}}} & 0 & 0 \\ 0 & 0 & \frac{1-k_{oo}}{2\sqrt{2k_{oo}}} & -\frac{1-k_{oo}}{2\sqrt{2k_{oo}}} \\ \frac{1-k_{oe}}{2\sqrt{2k_{oe}}} & \frac{1-k_{oe}}{2\sqrt{2k_{oe}}} & 0 & 0 \\ 0 & 0 & \frac{1-k_{oe}}{2\sqrt{2k_{oe}}} & \frac{1-k_{oe}}{2\sqrt{2k_{oe}}} \end{pmatrix}.$$

The coupling coefficients k_{oo}, and k_{oe} are defined from

$$Z_d/2 = Z_{oo} = k_{oo} Z_0, \quad 2Z_c = Z_{oe} = k_{oe} Z_0,$$

where Z_0 is the reference impedance, e.g., $50\,\Omega$; Z_{oo} and Z_{oe} are odd- and even-mode characteristic impedances. The values of the coupling coefficients k_{oo} and k_{oe} depend on how much signal coupling is present in a differential network [3] and $0 \leq k_{oo} \leq 1$, $k_{oe} \geq 1$.

Since the conversion formula (1) involves the inversion of matrix $T = M_2 S + M_1$, we would like to analyze the accuracy of such inversion, which we do by analyzing the condition number of this matrix. When there is no coupling, M_2 is the zero matrix, while M_1 is an orthogonal matrix, so matrix T is well-conditioned with the condition number $cond(T) = 1$. When there is coupling, the conversion matrix T depends on both M_1 and M_2 as well as S-parameter matrix S. While matrices M_1 and M_2 are orthogonal, nothing is known about matrix S, and, hence, about $T = M_2 S + M_1$. So, our goal is to analyze the condition number of this matrix in the presence of coupling. If the condition number is large, matrix T is ill-conditioned and direct inversion of this matrix may be highly unstable, which would cause inaccuracy in conversion between single-ended and mixed-mode S-parameters. In such situations, direct inversion of T should be avoided and the mixed-mode S parameters S^{mm} should be computed as a linear system

$$S^{mm}(M_2 S + M_1) = (M_1 S + M_2)$$

using, for example, the truncated SVD method to regularize the problem [8].

III. Broadside Coupled Striplines

As a model problem, we consider broadside coupled striplines depicted in Fig. 2. The conductors are 80 percent conductivity relative to the International Annealed Copper Standard (IACS). The dielectric is FR-4 with the relative permittivity chosen to be $\epsilon_r = 4.5$; for this example we have chosen w_0 to be 0, conductor width w, substrate thickness H, and conductor spacing s. This type of transmission lines is

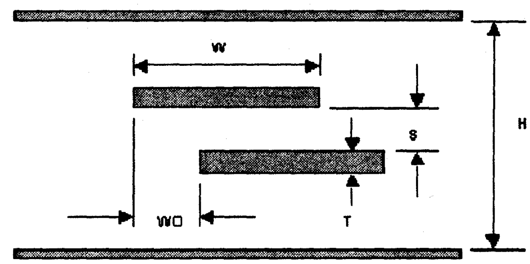

Fig. 2. Geometry of broadside coupled striplines.

used for analysis because the closed-form formula can be applied for a uniform dielectric material. The approach we used here follows the conversion procedure reported in [3]: first calculate the odd- and even-mode characteristic impedance to find coupling constants k_{oo} and k_{oe}, then construct the matrix M_1 and M_2. The single-ended S-parameters are approximated as follows: $|S_{11}| = |S_{22}| = |S_{33}| = |S_{44}|$ are approximated as reflection coefficients, $S_{14} = S_{23}$ are usually very small, i.e. 10^{-40}, $S_{12} = S_{12}$ and equal to coupling coefficient [9, p. 354], and $S_{12} = \pm\sqrt{1 - S_{11}^2 - S_{13}^2 - S_{14}^2}$ due to the conservation of energy. To simplify the problem, S-parameters are assumed to have zero phase to avoid the complexity of frequency dependence.

The condition number of matrix T as a function of spacing s between traces for different width w is shown in Fig. 3 using solid lines, for the case when $H = 20$ mils. Superimposed are extrapolations of $\mathrm{cond}(T)$, shown by dashed lines, as s approaches 0. Extrapolations were obtained by assuming that $\mathrm{cond}(T) = C \cdot s^{-p}$. Taking logarithm of both sides, we obtain $\ln \mathrm{cond}(T) = \ln C - p \ln s$, i.e. $\ln \mathrm{cond}(T)$ is a linear function of $\ln s$ with the slope $-p$. Constants C and p were obtained by the least square fitting of $\ln \mathrm{cond}(T)$ to a linear function. We find that $\mathrm{cond}(T) \sim s^{-3/2}$ for small s. As extrapolated curves indicate, as the differential traces come close to each other, or the spacing s approaches 0, the condition number of T increases reaching the values of the order of 10^{35}, which can affect the accuracy of conversion between single-ended and mixed mode S-parameters even using a double precision arithmetic available on most of computers.

It should be noted that in practice, single ended S-parameters may be accurate within only $10^{-7}-10^{-6}$ or even $10^{-3}-10^{-2}$, when data come from numerical simulations or measurements, respectively. In these cases, cond(T) of magnitude of a few orders may be too high to ensure that the converted mixed mode S parameters have the same order of accuracy.

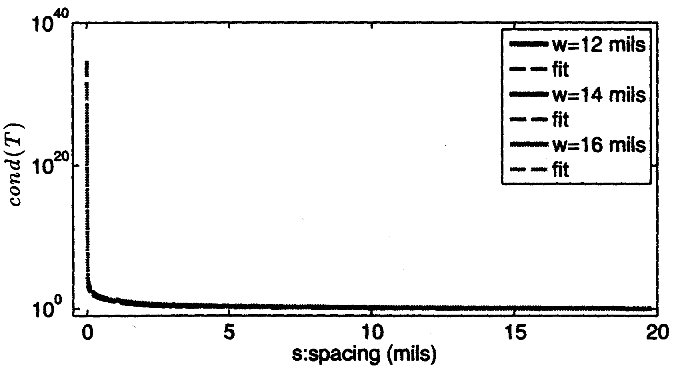

Fig. 3. cond(T) as a function of s at different w in the case of broadside coupled striplines. Superimposed are extrapolations of cond(T) modeled as $C \cdot s^{-p}$ with $p \approx 3/2$.

IV. Coupled Microstrip Pair

We consider the problem of coupled microstrip pair depicted in Fig. 4. For this type of transmission lines we chose the relative permittivity ϵ_r to be 4.5, substrate thickness H to be 20 mils, conductor thickness T to be 1.3 mils, the width of conductors w to vary from 0 to 20 mils, and the conductor spacing s to vary from 0.01645 to 20 mils. S parameters were approximated using a similar approach as for broadside coupled striplines while the closed-form formulae for approximating single-ended, odd-, and even- mode characteristic impedances Z_0, Z_{oo} and Z_{oe}, respectively, which are used in this analysis are from [10].

The behavior of cond(T) as a function of spacing s for different values of conductor width w is shown in Fig. 5 using solid lines. We also constructed extrapolated curves using a model cond$(T) = C \cdot s^{-p}$ as in the previous case. In this case, we find that $p \approx 0.4$, which gives slower growth of cond(T) as $s \to 0$ than in the case with broadside coupled striplines. Still, the results show that as $s \to 0$, cond(T) increases reaching the values of the order of 10^{10}, suggesting that direct inversion of T may not be accurate causing potentially large errors in mixed mode S-parameters especially when single ended S-parameters do not have high accuracy as it happens in measurements.

In order to provide an FEM EM simulation example, the model was simulated using ANSYS HFSS. We choose the relative permittivity to be $\epsilon_r = 4.5$, substrate thickness H to be 20 mils, conductor thickness T to be 1.3 mils, the width of microstrips w to be 4 mils, and conductor spacing s to be 0.015 mils. We use the PCB Impedance Calculator offered on *www.eeweb.com* which is based on the work of [10]. The odd- and even-mode characteristic impedances are 0.00273 and 183 Ohm, respectively to obtain single-ended S-parameters. In this case we use $Z_{oo} = 0.00273$ Ohm and $Z_{oe} = 183$ Ohm. The coupling constants k_{oo} and k_{oe} are computed as

$$k_{0o} = \sqrt{\frac{Z_{0o}}{Z_{0e}}}, \quad k_{0e} = \frac{1}{k_{0o}}.$$

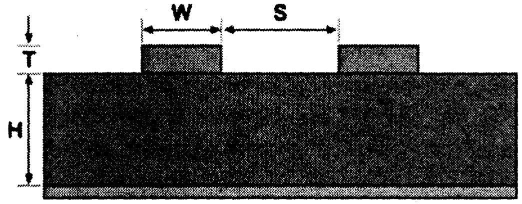

Fig. 4. Geometry of coupled microstrips.

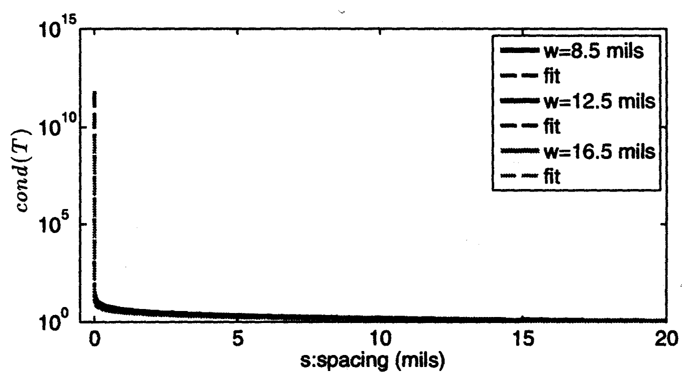

Fig. 5. cond(T) as a function of s at different w in the case of a coupled microstrip pair. Superimposed are extrapolations using a model cond$(T) \approx C \cdot s^{-p}$, where $p \approx 0.4$.

For completeness, in Fig. 6, we show the behavior of cond(T) as a function of frequency for the above parameters with $s = 0.015$ mils and $w = 4$ mils. As can be seen, cond(T) increases as frequency decreases, in particular, it becomes very large for low frequencies.

V. Conclusion

We analyzed stability of conversion between single-ended and mixed-mode S-parameters, proposed in [3]. The transformation matrix that needs to be inverted depends on S-parameters and may become ill-conditioned for some values of physical parameters leading to inaccurate results. We use two examples, broadside coupled striplines and coupled microstrip pairs, and show that when the spacing between traces becomes small, the condition number of the transformation matrix increases. For high condition numbers, a direct inversion should be avoided and methods for rank-deficient and ill-posed

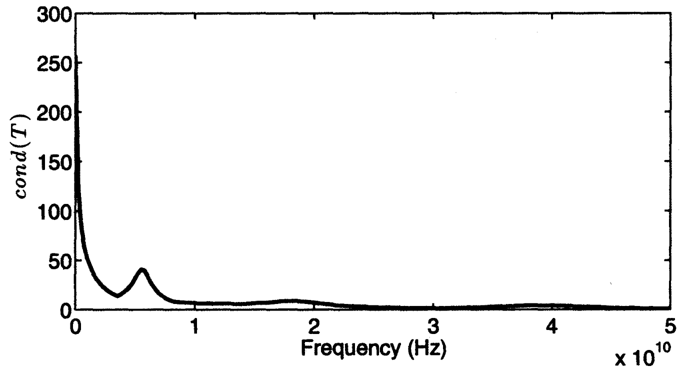

Fig. 6. $\mathrm{cond}(T)$ as a function of frequency in the case of a coupled microstrip pair.

problems such as the truncated singular value decomposition, should be used.

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