

## Euler Constant Slope Method

EXAMPLE

Solve approximately

$$\frac{dy}{dx} = \frac{x^2 + xy}{x + 2y}, \quad y(1) = 2.$$

Find  $y(2)$  in four steps.

$$h = \frac{2 - 1}{4} = .25$$

$$x_0 = 1, \quad y_0 = 2, \quad h = .25, \quad N = 4$$

$$x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + \left( \frac{x_n^2 + x_n y_n}{x_n + 2y_n} \right) h.$$

I do this on a hand help calculator and roundoff results to 3 decimal places

$$x_1 = 1 + .25 = \boxed{1.250}$$

$$y_1 = y_0 + \left( \frac{x_0^2 + x_0 y_0}{x_0 + 2y_0} \right) .25 = 2 + \left( \frac{1^2 + 1(2)}{1 + 2(2)} \right) .25 = 2 + \underbrace{(.6)}_{k_0} (.25) = \boxed{2.150}$$

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$$x_2 = 1.25 + .25 = \boxed{1.500}$$

$$y_2 = y_1 + \left( \frac{x_1^2 + x_1 y_1}{x_1 + 2y_1} \right) .25 = 2.150 + \left( \frac{(1.25)^2 + (1.25)(2.15)}{1.25 + 2(2.15)} \right) .25 \\ = 2.150 + \underbrace{(.765)}_{k_1} (.25) = \boxed{2.341}$$

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$$x_3 = \boxed{1.75}$$

$$y_3 = 2.341 + \left( \frac{(1.5)^2 + (1.5)(2.341)}{1.5 + 2(2.341)} \right) .25 = 2.341 + \underbrace{(.931)}_{k_2} (.25) = \boxed{2.573}$$

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$$x_4 = \boxed{2.00}$$

$$y_4 = 2.573 + \left( \frac{(1.75)^2 + (1.75)(2.573)}{1.75 + 2(2.573)} \right) .25 = 2.573 + \underbrace{(1.097)}_{k_3} (.25) = \boxed{2.847}$$