

## Improved Euler Method

### EXAMPLE

Use the improved Euler method to solve

$$y' = (x + y - 1)^2, \quad y(0) = 2.$$

Find  $y(.2)$  in 2 steps. Here  $h = .1$ , thus

$$u_1 = y_0 + (x_0 + y_0 - 1)^2 h = 2 + (0 + 2 - 1)^2 .1 = 2.100; \quad x_1 = 0 + .1 = \boxed{.1}$$

$$\begin{aligned} y_1 &= y_0 + \frac{1}{2} \{ (x_0 + y_0 - 1)^2 + (x_1 + u_1 - 1)^2 \} h \\ &= 2 + \frac{1}{2} \{ (0 + 2 - 1)^2 + (.1 + 2.1 - 1)^2 \} .1 = \boxed{2.122} \end{aligned}$$

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$$u_2 = y_1 + (x_1 + y_1 - 1)^2 h = 2.122 + (.1 + 2.122 - 1)^2 .1 = 2.271; \quad x_2 = \boxed{.2}$$

$$\begin{aligned} y_2 &= y_1 + \frac{1}{2} \{ (x_1 + y_1 - 1)^2 + (x_2 + u_2 - 1)^2 \} h \\ &= 2.122 + \frac{1}{2} \{ (.1 + 2.122 - 1)^2 + (.2 + 2.271 - 1)^2 \} .1 = \boxed{2.305} \end{aligned}$$