LU factorization for a tridiagonal matrix $A_h u = f$

$$\begin{pmatrix} b_1 & c_1 & & & \\ a_2 & b_2 & c_2 & & 0 \\ & \ddots & \ddots & \ddots & \\ & 0 & a_{n-1} & b_{n-1} & c_{n-1} \\ & & & a_n & b_n \end{pmatrix} =$$

$$\begin{pmatrix} 1 & & & & \\ l_2 & 1 & & 0 & \\ & l_3 & 1 & & \\ & & \ddots & \ddots & \\ & 0 & & l_n & 1 \end{pmatrix} \begin{pmatrix} v_1 & c_1 & & & \\ & v_2 & c_2 & & 0 \\ & & \ddots & \ddots & \\ & & & v_{n-1} & c_{n-1} \\ & 0 & & & v_n \end{pmatrix}$$

<u>To determine L, U:</u>

$$\begin{aligned} b_1 = v_1 \quad &\Rightarrow \quad v_1 = b_1 \\ a_k = l_k v_{k-1} \quad &\Rightarrow \quad l_k = a_k / v_{k-1} \\ b_k = l_k c_{k-1} + v_k \quad &\Rightarrow \quad v_k = b_k - l_k c_{k-1}, \quad k = 2, \ldots, n \end{aligned}$$

<u>To solve $Ly = f$:</u>

$$\begin{aligned} y_1 &= f_1 \\ l_k y_{k-1} + y_k = f_k \quad &\Rightarrow \quad y_k = f_k - l_k y_{k-1}, \quad k = 2, \ldots, n \end{aligned}$$

<u>To solve $Uu = y$:</u>

$$\begin{aligned} v_n u_n = y_n \quad &\Rightarrow \quad u_n = y_n / v_n \\ v_k u_k + c_k u_{k+1} = y_k \quad &\Rightarrow \quad u_k = (y_k - c_k u_{k+1}) / v_k, \quad k = n-1, \ldots, 1 \end{aligned}$$

<u>operation count</u>: # of multiplications $\sim 3n \ll \frac{n^3}{3}$