Math 175: Exam 1 review

Note: exam #1 covers sections 7.5, 7.6, 8.1, 8.2, 4.7

In class we evaluated $\int \tan^4 x\, dx =$

$$= \int \tan^2 x \cdot \underbrace{\tan^2 x}_{1-\sec^2 x}\, dx = \int \tan^2 x (1-\sec^2 x)\, dx =$$

$$= \int \tan^2 x\, dx - \int \tan^2 x \cdot \sec^2 x\, dx =$$

$$= \int (1-\sec^2 x)\, dx - \underbrace{\int \tan^2 x \cdot \sec^2 x\, dx}_{u=\tan x} = \dots$$

Ex $$\int \tan^6 x\, dx = \int \tan^4 x \cdot \underbrace{\tan^2 x}_{1-\sec^2 x}\, dx =$$

$$= \underbrace{\int \tan^4 x\, dx}_{\text{done in class}} - \underbrace{\int \tan^4 x \cdot \sec^2 x\, dx}_{u=\tan x} = \dots$$

Another approach is to use reduction formulas.

Note The test problems would need to be solved without reduction formulas but using trig. identities and substitutions.

Ex $\int \sin^5(8x)\,dx = \int \sin^4(8x)\cdot \underbrace{\sin(8x)\,dx}_{\sim du} \circledequal$

5: odd

$\sin^4(8x) \to (\sin^2(8x))^2 \to (1-\cos^2(8x))^2$

Use $u=\cos(8x)$, $du=-8\sin(8x)$

$\circledequal \int (1-u^2)^2\left(-\frac{1}{8}\right)du = -\frac{1}{8}\int (1-2u^2+u^4)\,du$

$= \dots$

Ex $\int_0^{\pi/4} \tan^4\theta \underbrace{\sec^2\theta\, d\theta}_{du} = \left| \begin{array}{l} u=\tan\theta \\ du=\sec^2\theta\, d\theta \end{array} \right| =$

$\theta=0 \Rightarrow u=0$

$\theta=\frac{\pi}{4} \Rightarrow u=1$

$= \int_0^1 u^4\,du = \frac{u^5}{5}\Big|_0^1 = \frac{1}{5}$

L'Hopital Rule

#15 $\lim_{x\to 0} (x+\cos x)^{\frac{1}{x}} = 1^\infty$

S7.6

$(x+\cos x)^{\frac{1}{x}} = e^{\ln(x+\cos x)^{\frac{1}{x}}} = e^{\frac{1}{x}\ln(x+\cos x)}$

$\lim_{x\to 0} (x+\cos x)^{\frac{1}{x}} = \lim_{x\to 0} e^{\frac{1}{x}\ln(x+\cos x)} =$

$$= e^{\lim_{x\to 0} \frac{1}{x}\ln(x+\cos x)} \qquad = e^{L}$$

where

$$L = \lim_{x\to 0} \underbrace{\frac{1}{x}\ln(x+\cos x)}_{\infty\,\cdot\,0} = \lim_{x\to 0} \frac{\ln(x+\cos x)}{x} = \frac{0}{0}$$

$$\overset{\text{l'Hop.}}{\underset{\text{rule}}{=}} \lim_{x\to 0} \frac{\frac{1-\sin x}{x+\cos x}}{1} = \frac{1-0}{0+1} = \boxed{1 = L}$$

Hence,

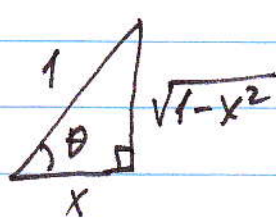
$$\lim_{x\to 0} (x+\cos x)^{\frac{1}{x}} = e^{L} = e^{1} = e.$$

Inverse trigonometric functions

#34 §7.5

Evaluate $\tan(\cos^{-1}x) =$

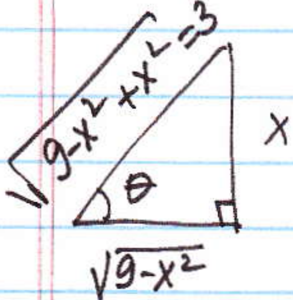
Let $\theta = \cos^{-1}x \Rightarrow \cos\theta = x$



$$= \tan\theta = \frac{\sqrt{1-x^2}}{x}$$

(#38) $\cos\left(\tan^{-1}\left(\frac{x}{\sqrt{9-x^2}}\right)\right) \ominus$

Let $\theta = \tan^{-1}\left(\frac{x}{\sqrt{9-x^2}}\right) \Rightarrow \tan\theta = \frac{x}{\sqrt{9-x^2}}$



$\ominus \cos\theta = \frac{\sqrt{9-x^2}}{3}$

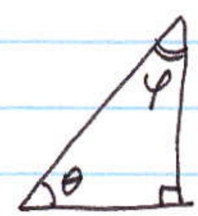
(#31) Evaluate $\csc^{-1}(\sec 2)$

Let $x = \sec 2 \Rightarrow \sec^{-1} x = 2$?

Recall $\csc^{-1}(\csc(y)) = y$, $|y| \geq 1$

$\sec^{-1}(\sec(y)) = y$, $|y| \geq 1$

Recall identity $\theta + \varphi = \frac{\pi}{2}$



or $\boxed{\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}}$

There is another identity:

$\boxed{\sec^{-1} x + \csc^{-1} x = \frac{\pi}{2}}$

$$\Rightarrow \quad \csc^{-1} x = \frac{\pi}{2} - \sec^{-1} x$$

$$\csc^{-1}(\sec 2) = \frac{\pi}{2} - \underbrace{\sec^{-1}(\sec 2)}_{= 2} = \frac{\pi}{2} - 2$$

Integration by parts

#34 S8.1

$$\int_0^{1/\sqrt{2}} y \tan^{-1} y^2 \, dy = \left| \begin{array}{ll} u = \tan^{-1} y^2 & dv = y\,dy \\ du = \dfrac{2y\,dy}{1+y^4} & v = \dfrac{y^2}{2} \end{array} \right| =$$

$$= \frac{y^2}{2} \tan^{-1} y^2 \Big|_0^{\frac{1}{\sqrt{2}}} - \int_0^{1/\sqrt{2}} \frac{y^2}{2} \cdot \frac{2y}{1+y^4}\, dy =$$

$$= \frac{1}{2} \cdot \frac{1}{2} \tan^{-1} \frac{1}{2} - \int_0^{1/\sqrt{2}} \frac{y^3\,dy}{1+y^4} = \left| \begin{array}{l} u = 1+y^4 \\ du = 4y^3\,dy \\ y=0 \Rightarrow u = 1 \\ y = \frac{1}{\sqrt{2}} \Rightarrow u = \underbrace{5/4}_{} \end{array} \right|$$

$$1 + \left(\frac{1}{\sqrt{2}}\right)^4 = 1 + \frac{1}{2^2} = 1 + \frac{1}{4}$$

$$= \frac{1}{4} \tan^{-1} \frac{1}{2} - \int_1^{5/4} \frac{\frac{1}{4}\,du}{u} =$$

$$= \frac{1}{4} \tan^{-1} \frac{1}{2} - \frac{1}{4} \ln u \Big|_1^{5/4} =$$

$$= \frac{1}{4} \tan^{-1} \frac{1}{2} - \frac{1}{4} \ln \frac{5}{4}$$

Compare growth rates

#83 S 7.6 Compare $\sqrt{x}$ and $\ln^{10} x$ as $x \to \infty$

$$\lim_{x\to\infty} \frac{\sqrt{x}}{\ln^{10} x}$$

$$\left(\frac{\sqrt{x}}{\ln^{10} x}\right)^2 = \frac{x}{\ln^{20} x}$$

$$\lim_{x\to\infty} \frac{x}{\ln^{20} x} = \frac{\infty}{\infty} \overset{\text{l'Hop.}}{\underset{\text{rule}}{=}} \lim_{x\to\infty} \frac{1}{20 \ln^{19} x \cdot \frac{1}{x}} =$$

$$= \lim_{x\to\infty} \frac{x}{20 \ln^{19} x} \overset{\text{l'Hop.}}{\underset{\text{rule}}{=}} \lim_{x\to\infty} \frac{x}{20.19 \ln^{18} x} \overset{\text{l'Hop.}}{=} \dots$$

$$= \lim_{x\to\infty} \frac{x}{20.19.18 \dots 2 \ln x} \overset{\text{l'Hop.}}{=} \lim_{x\to\infty} \frac{1}{20.19 \dots 2.1 \frac{1}{x}} \; =$$

$20.19.18 \dots 2.1 = 20!$ factorial

$n! = 1 \cdot 2 \cdot 3 \dots \cdot (n-1)\, n$

$$= \lim_{x\to\infty} \frac{x}{20!} = \infty \Rightarrow x >> \ln^{20} x$$

$$\therefore \sqrt{x} >> \ln^{10} x$$