3/5/2014

Math 175: Exam #2

8.3 Trigonom. substitutions

$a^2 - x^2$ $\quad x = a\sin\theta, \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$a^2 + x^2$ $\quad x = a\tan\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$x^2 - a^2$ $\quad x = a\sec\theta \quad \begin{cases} 0 \le \theta < \frac{\pi}{2}, & x \ge a \\ \frac{\pi}{2} < \theta \le \pi, & x \le -a \end{cases}$

8.4 Partial fraction decomposition

- to be able to set up partial fraction decomposition and evaluate coefficients if necessary

8.6 Numerical Integration

Trapezoid Rule:

$$\int_a^b f(x)\,dx = \left(\frac{1}{2} f(x_0) + f(x_1) + \ldots + f(x_{n-1}) + \frac{1}{2} f(x_n)\right)\Delta x + \underbrace{C_T (\Delta x)^2}_{\text{error}_{\Delta x}}$$

$$\Delta x \to \frac{\Delta x}{2} \Rightarrow \text{error} \to \underset{\left(\frac{1}{2}\right)^2}{\frac{1}{4}} \text{error}$$

Midpoint Rule:

$$\int_a^b f(x)\,dx = \sum_{k=1}^{n} f(m_k)\,\Delta x + C_M \cdot (\Delta x)^2$$

$$m_k = \frac{x_{k-1} + x_k}{2}$$

Simpson's Rule

$$\int_a^b f(x)\,dx = \Big(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \ldots$$

$$+ \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\Big)\frac{\Delta x}{3} + C_S \cdot (\Delta x)^4$$

n : even

$$\Delta x \to \frac{\Delta x}{2} \Rightarrow \text{error} \to \left(\frac{1}{2}\right)^4 \text{error}$$

- need to be able to compute an approximation to $\int_a^b f(x)\,dx$ with fixed n, a, b, and given $f(x)$
- know how error depends on power Δx^p
  $p = 2$ or 4

8.7 Improper Integrals

- Know def of improper integrals and how to evaluate them.

8.8 ODEs

→ - separable DEs
- find a general solution (in terms of an arbitrary constant)
- use initial condition to find particular solution

9.1, 9.2 Sequences

- Def of a sequence
→ - recurrence formula
→ - explicit formula
→ - limit of a sequence
- monotonic, bounded sequences
- Thm about convergence of a monotonic bounded sequence
- Squeeze Thm
- Growth rates seq. $\{a_n\} = \{r^n\}$: convergence

9.1, 9.3. Series

- Def
- sum of a series
- geometric series $\sum_{n=0}^{\infty} ar^n$
- telescoping series

<u>Ex</u> $\frac{dy}{dt} = ky$: separable DE

$y \neq 0$ $\quad \frac{dy}{y} = k\,dt$

$$\int \frac{dy}{y} = \int k\,dt$$

exp / $\ln|y| = kt + \tilde{C}$ $\qquad e^{\ln x} = x$

$$|y| = e^{kt + \tilde{C}} = e^{kt} \cdot e^{\tilde{C}}$$

or $\boxed{y(t) = C e^{kt}}$: <u>the general solution</u> of

$$\frac{dy}{dt} = ky$$

1st order DE has a general solution that depends on one arbitrary constant C

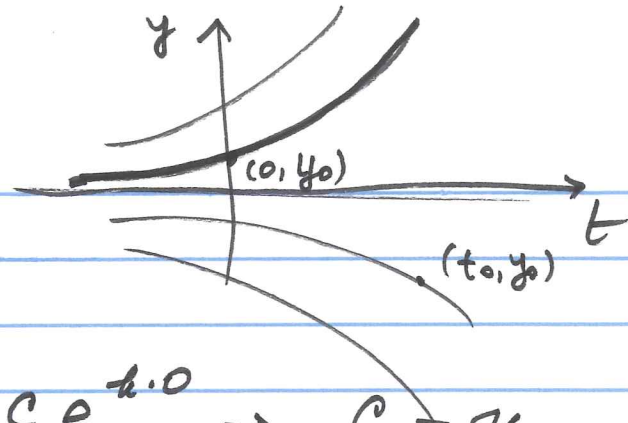
n^{th} order DE has a general solution that depends on n arbitrary constants.

Consider initial value problem (IVP)

$$\underbrace{\frac{dy}{dt} = ky}_{DE}, \qquad \underbrace{y(0) = y_0}_{\text{initial condition (IC)}}$$

IVP = DE + IC

$y(t) = Ce^{kt}$



$y(0) = y_0$

at $t=0$: $\underbrace{y(0)}_{= y_0} = C\underbrace{e^{k\cdot 0}}_{= 1} \Rightarrow C = y_0$

$\boxed{y(t) = y_0 e^{kt}}$: <u>particular solution</u>

#10 S 8.8

$\frac{dy}{dt} = 8e^{-4t} + 1, \quad y(0) = 5$

$y' = f(t) \Rightarrow y = \int f(t)\,dt$

$y = \int \frac{dy}{dt}\,dt = \int (8e^{-4t} + 1)\,dt =$

$y(t) = -2e^{-4t} + t + C$: the <u>general solution</u>

To find C we use IC $y(0) = 5$

$5 = y(0) = -2e^{-4\cdot 0} + 0 + C \Rightarrow C = 7$

$\Rightarrow \boxed{y(t) = -2e^{-4t} + t + 7}$: particular solution

#23 S8.8

$$\frac{dy}{dt} = \frac{3t^2}{y} \text{ : separable DE}$$

$$y\,dy = 3t^2\,dt$$

$$\int y\,dy = \int 3t^2\,dt$$

$$\frac{y^2}{2} = t^3 + C$$

$$y^2 = 2(t^3 + C)$$

$$y = \pm\sqrt{2(t^3+C)}$$

if $y(t_0) = y_0$ and $y_0 > 0 \Rightarrow y = \sqrt{2(t^3+C)}$

$y_0 \le 0 \Rightarrow y = -\sqrt{2(t^3+C)}$

Ex

$$\frac{dy}{dx} = 4x^3y - y$$

$$\frac{dy}{dx} = (4x^3 - 1)y \text{ : separable DE}$$

$$\frac{dy}{y} = (4x^3 - 1)\,dx$$

. . . .

Ex

$$x\frac{dy}{dx} - y = 2x^2y$$

$$x\frac{dy}{dx} = y + 2x^2y$$

$$x\frac{dy}{dx} = y(1+2x^2) \quad : \text{ separable DE}$$

$$\frac{dy}{y} = \frac{1+2x^2}{x}\,dx$$

$$\int \frac{dy}{y} = \int \frac{1+2x^2}{x}\,dx$$

$$\int \frac{dy}{y} = \int \frac{dx}{x} + 2\int x\,dx$$

. . . .

#27 $\frac{dy}{dt} = ty + 2$: nonseparable DE

#25 S8.3 $\int \frac{\sqrt{x^2-9}}{x}\,dx, \quad x>3$

$\sin^2\theta + \cos^2\theta = 1$

$1+\tan^2\theta = \sec^2\theta$

$a=3$ $\boxed{x = 3\sec\theta}$

$dx = 3\sec\theta\tan\theta\,d\theta$

$$x^2 - 9 = 9\sec^2\theta - 9 = 9(\sec^2\theta - 1) = 9\tan^2\theta$$

$$\int \frac{\sqrt{x^2-9}}{x}\,dx = \int \frac{\sqrt{9\tan^2\theta}}{\cancel{3\sec\theta}} \cdot \cancel{3\sec\theta}\tan\theta\,d\theta =$$

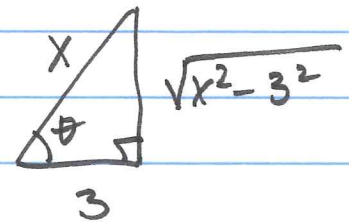
$$= 3\int \tan^2\theta\,d\theta = 3\int(\sec^2\theta - 1)\,d\theta =$$

$$= 3(\tan\theta - \theta) + C \quad =$$

$$\tan\theta = \sqrt{\sec^2\theta - 1} = \sqrt{\left(\frac{x}{3}\right)^2 - 1} = \frac{\sqrt{x^2-9}}{3}$$

$$x = 3\sec\theta \Rightarrow \sec\theta = \frac{x}{3}$$

or $\cos\theta = \frac{3}{x}$



$$\therefore \tan\theta = \frac{\sqrt{x^2-9}}{3} \quad \checkmark$$

$$\textcircled{=}\ 3\left(\frac{\sqrt{x^2-9}}{3} - \underset{\sec^{-1}}{\operatorname{arcsec}}\frac{x}{3}\right) + C =$$

$$= \sqrt{x^2-9} - 3\sec^{-1}\frac{x}{3} + C$$

Ex

$$\frac{1+2r}{1-r^2}$$

$$1 + 2r + r^2 + 2r^3 + r^4 + 2r^5 + r^6 + \ldots = \qquad |r|<1$$

$$= (1 + r^2 + r^4 + r^6 + \ldots) + (2r + 2r^3 + 2r^5 + \ldots)$$

$$= (1 + r^2 + r^4 + r^6 + \ldots) + 2r(1 + r^2 + r^4 + \ldots) \quad \textcircled{=}$$

$$1 + r^2 + r^4 + r^6 + \ldots = \left| \begin{array}{l} R = r^2 \\ |r|<1 \Rightarrow |R|<1 \end{array} \right| =$$

$$= 1 + R + R^2 + \ldots = \frac{1}{1-R} = \frac{1}{1-r^2}$$

geom. series $a = 1$, ratio R

$$= (1+2r)\underbrace{(1+r^2+r^4+\dots)}_{\frac{1}{1-r^2}} = \frac{1+2r}{1-r^2}$$

#30
§8.4

$$\frac{x^2+2}{x\underbrace{(x^2+5x+8)}_{\text{irreducible}}} = \frac{A}{x} + \frac{Bx+C}{x^2+5x+8}$$

$$\frac{-5 \pm \sqrt{5^2-4\cdot 8}}{2}$$ complex roots

#29
§8.4

$$\frac{2x^2+3}{(x^2-8x+16)(x^2+3x+4)} = \frac{2x^2+3}{(x-4)^2\underbrace{(x^2+3x+4)}_{\text{irreducible}}} =$$

$$\frac{-3 \pm \sqrt{3^2-4\cdot 4}}{2}$$

$$= \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{Cx+D}{x^2+3x+4}$$