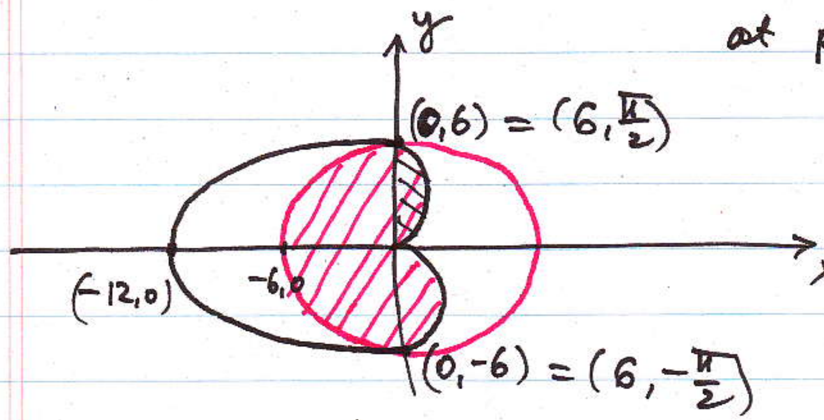
4/25/2014

Math 175 : review # 4.

Ex Find the area of the region inside the circle $r = 6$ and the cardiod $r = 6(1-\cos\theta)$.



at pts (0,6), (0,-6)

$r = 6$:

$6(1-\cos\theta) = 6$

$\rightarrow \cos\theta = 0$

$\Downarrow$

$\theta = \pm\frac{\pi}{2}$

$$A_1 = \text{Area} = \int_0^{\pi/2} \frac{1}{2}\, 6^2 (1-\cos\theta)^2\, d\theta =$$

$r = f(\theta)$

$\int_\alpha^\beta \frac{1}{2} (f(\theta))^2 d\theta$

$f(\theta) = 6(1-\cos\theta)$

$$= \frac{6^2}{2} \int_0^{\pi/2} (1 - 2\cos\theta + \cos^2\theta)\, d\theta =$$

$$= \frac{6^2}{2} \int_0^{\pi/2} \left(1 - 2\cos\theta + \frac{1+\cos 2\theta}{2}\right) d\theta =$$

$$= \frac{6^2}{2} \int_0^{\pi/2} \left(\frac{3}{2} - 2\cos\theta + \frac{1}{2}\cos 2\theta\right) d\theta =$$

$$= \frac{6^2}{2} \left(\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin 2\theta\right)\Big|_0^{\pi/2} =$$

$$= \frac{6^2}{2}\left(\frac{3}{2}\cdot\frac{\pi}{2} - 2\underbrace{\sin\frac{\pi}{2}}_{1}\right) = \frac{6^2}{2}\left(\frac{3\pi}{4} - 2\right)$$

area of common region $= 2A_1 + \underbrace{\frac{1}{2}\pi \cdot 6^2}_{\text{area of circle of rad } r=6} =$

$= \cancel{2} \cdot \frac{6^2}{\cancel{2}}\left(\frac{3\pi}{4} - 2\right) + \frac{\pi \cdot 6^2}{2} = 6^2\left[\frac{3\pi}{4} - 2 + \frac{\pi}{2}\right] =$

$= 6^2\left(\frac{5\pi}{4} - 2\right) = \frac{6^2 \cdot 5\pi}{4} - 6^2 \cdot 2 = 45\pi - 72$

$\frac{3}{4} + \frac{1}{2}^{(2} = \frac{5}{4}$

$\frac{6^2}{4} = \frac{2^2 \cdot 3^2}{4} = 9$

Note: Two curves $r=6$ and $r=6(1-\cos\theta)$ intersect when

$6 = 6(1-\cos\theta) \Rightarrow \cos\theta = 0 \Rightarrow \theta = \pm\frac{\pi}{2}$

#13 S 11.3

Find the slope of the tangent line to the curve

$r^2 = 4\cos 2\theta$ at $\left(0, \pm\frac{\pi}{4}\right)$

$r = f(\theta)$

$\frac{dy}{dx}$: slope of tangent line

$x = r\cos\theta, \quad y = r\sin\theta$

$x = f(\theta)\cos\theta, \quad y = f(\theta)\sin\theta$

$$\frac{dy}{dx} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

implicit differentiation

$$\frac{d}{d\theta} \qquad r^2 = 4\cos 2\theta \qquad\qquad r = f(\theta) = 2\sqrt{\cos 2\theta}$$

$$2r \underbrace{\frac{dr}{d\theta}}_{f'(\theta)} = -4\sin 2\theta \cdot 2$$

$$\frac{dr}{d\theta} = f'(\theta) = \frac{-8\sin 2\theta}{2r} = \frac{-8\sin 2\theta}{2\cdot 2\sqrt{\cos 2\theta}}$$

$$= -4\frac{\sin 2\theta}{\sqrt{\cos 2\theta}}$$

$$f(\theta) = 2\sqrt{\cos 2\theta}$$

$$f'(\theta) = -4\frac{\sin 2\theta}{\sqrt{\cos 2\theta}}$$

$$\frac{dy}{dx} = \frac{-4\frac{\sin 2\theta}{\sqrt{\cos 2\theta}}\cdot\sin\theta + 2\sqrt{\cos 2\theta}\cdot\cos\theta}{-4\frac{\sin 2\theta}{\sqrt{\cos 2\theta}}\cos\theta - 2\sqrt{\cos 2\theta}\cdot\sin\theta} =$$

$$= \frac{-4\sin 2\theta\cdot\sin\theta + 2\cos 2\theta\cdot\cos\theta}{-4\sin 2\theta\cdot\cos\theta - 2\cos 2\theta\cdot\sin\theta}$$

$$\left.\frac{dy}{dx}\right|_{(0,\frac{\pi}{4})} = \left.\frac{f'(\theta)\sin\theta + \overbrace{f(\theta)}^{0}\cos\theta}{f'(\theta)\cos\theta - \underbrace{f(\theta)}_{0}\sin\theta}\right|_{\theta=\frac{\pi}{4}} = \left.\frac{\sin\theta}{\cos\theta}\right|_{\theta=\frac{\pi}{4}}$$

$$r=0 \Rightarrow f(\theta)\Big|_{\theta=\frac{\pi}{4}}=0 \qquad = \tan\theta\Big|_{\theta=\frac{\pi}{4}} = 1$$

$$\left.\frac{dy}{dx}\right|_{(0,-\frac{\pi}{4})} = \tan\left(-\frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -1$$

Tangent line w/ slope $\frac{dy}{dx} = k$ that goes through a pt (x_0, y_0) is

$$y - y_0 = k(x - x_0)$$

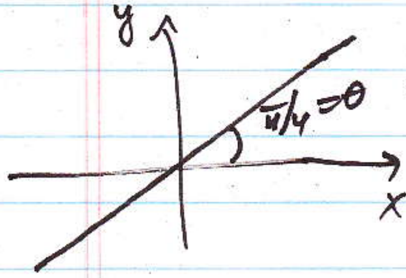
$$k = \frac{y-y_0}{x-x_0}$$

(x_0, y_0)

$r=0,\ \theta=\frac{\pi}{4}$ $\quad x = r\cos\theta = 0$ $\quad \Rightarrow (x_0, y_0) = (0,0)$

$\quad y = r\sin\theta = 0$

$$y - 0 = \underset{\overset{\parallel}{\frac{dy}{dx}}}{1}(x-0) \Rightarrow y = x$$



$r\sin\theta = r\cos\theta$

$\tan\theta = 1$

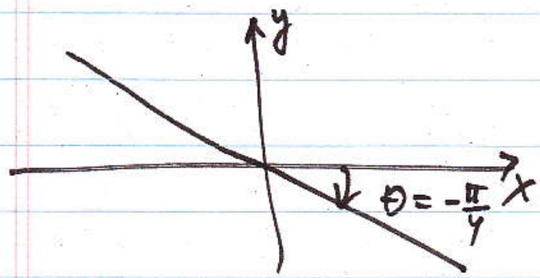
$y = x \iff \theta = \frac{\pi}{4}$

$r=0, \ \theta = -\frac{\pi}{4} \Rightarrow x_0 = y_0 = 0$

$y - 0 = -1(x-0) \Rightarrow y = -x$

or $\tan\theta = -1$

$\Rightarrow \theta = -\frac{\pi}{4}$



Taylor series

a: center

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots$$
$$\ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_n(x)$$

$P_n(x)$: Taylor polynomial of degree n

$$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-a)^{n+1}$$

ξ is between x and a

- Review how to convert equations between polar representation and Cartesian representation

$$x = r\cos\theta, \quad y = r\sin\theta$$

$$r^2 = x^2 + y^2, \quad \tan\theta = \frac{y}{x}$$

- Parametric curves, graphing
  slope of a tangent, eq^{n} of tangent line

- Polar coordinates

- Area of a region defined by curves in polar equations.

- Taylor and Maclaurin series ($a=0$)
  - definition
  - review Taylor series of $\sin x$, $\cos x$, e^x, $\ln(1+x)$, $\tan^{-1}x$, $\frac{1}{1-x}$ (geom. series)

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$ geom. series $|x|<1$

$$\ln(1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \dots\right)$$

- Binomial series

- Using differentiation, integration, addition, subtraction, multiplication by powers of x to get or recognize other series.