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Lecture 13

Math 275: Review #1

#35 S 12.6 Reduce equation to one of the standard forms, sketch traces, classify the surface and sketch it.

$$x^2 + y^2 - 2x - 6y - z + 10 = 0$$

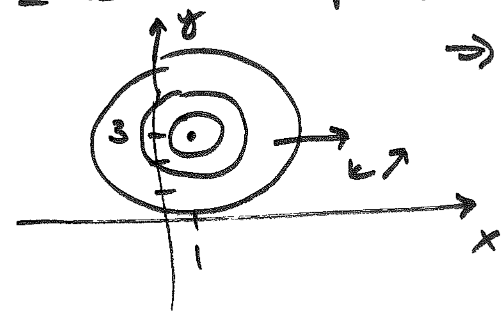
Idea: complete the square

$$(x^2 - 2x + 1) - 1 + (y^2 - 6y + 9) - 9 - z + 10 = 0$$

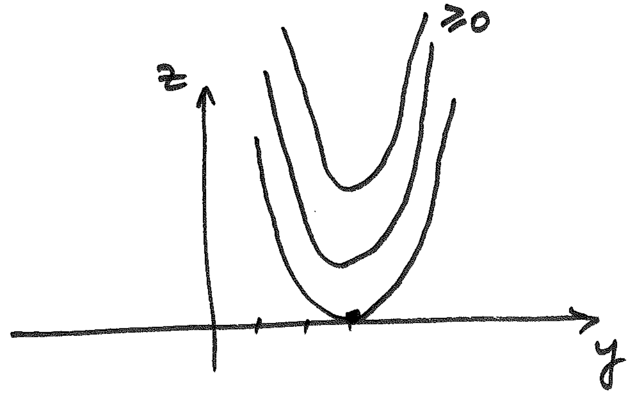
$$(x-1)^2 + (y-3)^2 - z = 0$$

Trace w/ $z = k$: $(x-1)^2 + (y-3)^2 = k$: ellipse / circle for $\overset{=z}{k} \geq 0$

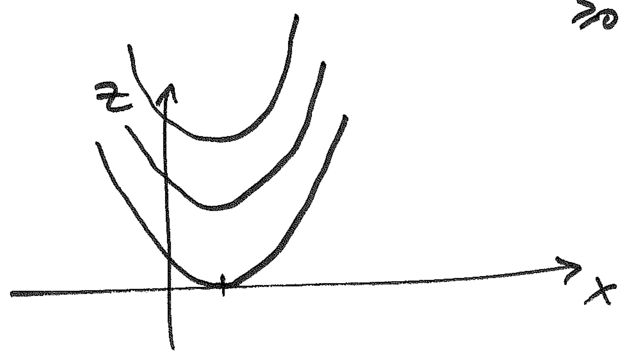
$\Rightarrow z \geq 0$



Trace w/ $x=k$: $z = (y-3)^2 + \underbrace{(k-1)^2}_{\geq 0}$: parabolas



Trace w/ $y=k$: $z = (x-1)^2 + \underbrace{(k-3)^2}_{\geq 0}$: parabolas



$\therefore$ traces are parabolas & ellipses $\Rightarrow$ we have elliptic paraboloid

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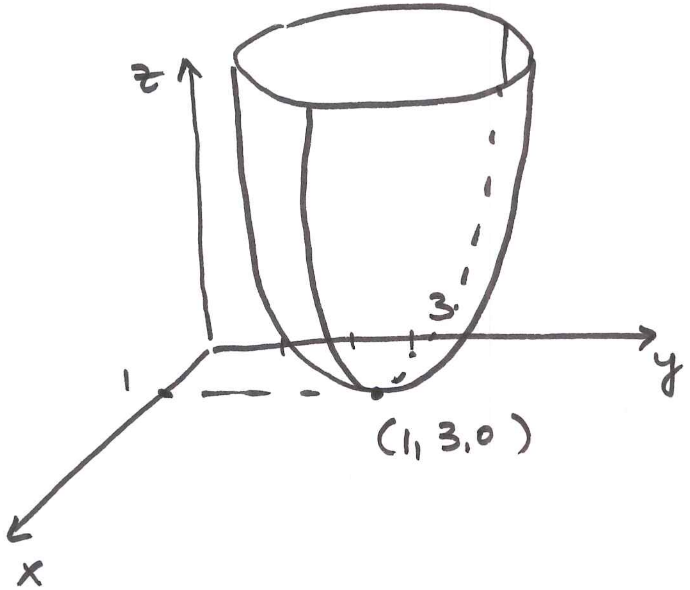


$$(x-1)^2+(y-3)^2-z=0$$

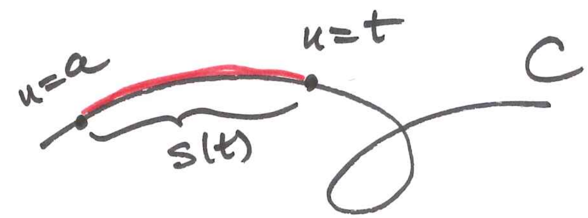
or

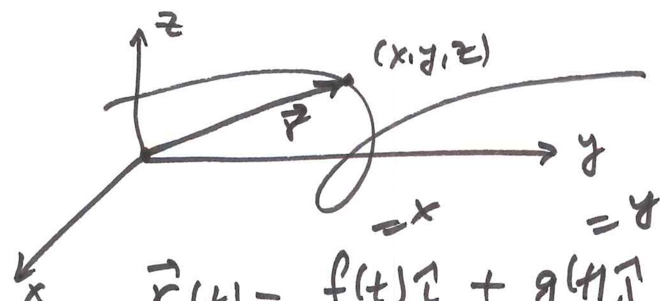
$$(x-1)^2+(y-3)^2=z$$

#13
S 13.3

(a) Find the arc length function for the curve measured from point P in the direction of increasing t and then reparametrize the curve wrt the arc length starting from P(4, 1, 3);

$$\vec{r}(t)=(5-t)\hat{\imath}+(4t-3)\hat{\jmath}+3t\,\hat{k}$$





$$\vec{r}(t)=\underbrace{f(t)}_{=x}\hat{\imath}+\underbrace{g(t)}_{=y}\hat{\jmath}+\underbrace{h(t)}_{=z}\hat{k}$$

$$s(t)=\int_a^t|\vec{r}\,'(u)|\,du$$

we find a from pt $P(4,1,3)$:

$$\left.\begin{aligned} x &= 5-t = 4 \\ y &= 4t-3 = 1 \\ z &= 3t = 3 \end{aligned}\right\} \Rightarrow \boxed{t=1} \Rightarrow \boxed{a=1}$$

$$\vec{r}\,'(t) = -\hat{\imath} + 4\hat{\jmath} + 3\hat{k} \Rightarrow |\vec{r}\,'(t)| = \sqrt{(-1)^2+4^2+3^2} = \sqrt{26}$$

$$\therefore\ s(t) = \int_1^t \sqrt{26}\,du = \sqrt{26}\cdot u\Big|_{u=1}^{u=t} = \sqrt{26}\,(t-1)$$

$$\Rightarrow \boxed{s = \sqrt{26}\,(t-1)}$$

To reparametrize the curve in terms of s, solve for t and substitute into $\vec{r}(t)$:

$$s = \sqrt{26}\,(t-1) \Rightarrow \boxed{t = 1 + \frac{s}{\sqrt{26}}}$$

$$\therefore \left\{ \begin{aligned} x(s) &= 5 - \underbrace{\left(1 + \frac{s}{\sqrt{26}}\right)}_{t} = 4 - \frac{s}{\sqrt{26}} \\ y(s) &= 4\cdot\left(1 + \frac{s}{\sqrt{26}}\right) - 3 = 1 + \frac{4s}{\sqrt{26}} \\ z(s) &= 3\left(1 + \frac{s}{\sqrt{26}}\right) \end{aligned} \right.$$

(b) find the point 4 units along the curve (in the direction of increasing t) from P.

at pt P: $t=1$

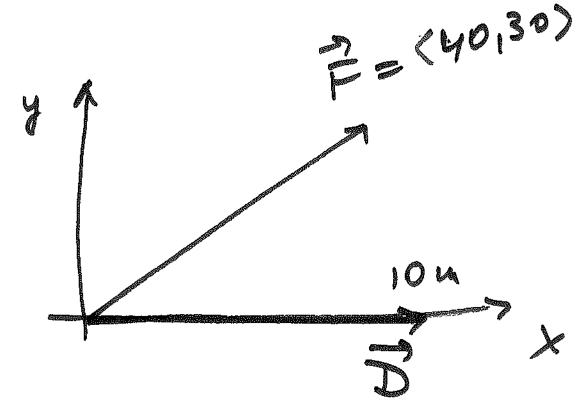
$\therefore$ 4 units from $t=1$ is when $t = 1+4 = 5$

$\Rightarrow$ new pt is $x = (5-t)\big|_{t=5} = 0$; $y = (4t-3)\big|_{t=5} = 17$

$z = 3t\big|_{t=5} = 15$, i.e. $\boxed{(0, 17, 15)}$

Ex a constant force $\vec{F} = \langle 40, 30 \rangle$ is used to move a sled horizontally 10 m. Calculate the work done.

Work $W = \vec{F} \cdot \vec{D}$ displacement vector



$\vec{D} = \langle 10, 0 \rangle$

$W = \vec{F} \cdot \vec{D} = \langle 40, 30 \rangle \cdot \langle 10, 0 \rangle = 40 \cdot 10 + 30 \cdot 0 = 400$ J

Ex Find line of intersection of planes $x - z = 1$ and $y + 2z = 3$

$x - z = 1$ $\vec{n}_1 = \langle 1, 0, -1 \rangle$

$y + 2z = 3$ $\vec{n}_2 = \langle 0, 1, 2 \rangle$

$\vec{n}_1 \not\parallel \vec{n}_2$ since $\vec{n}_1 \neq \underbrace{k}_{\text{const}} \cdot \vec{n}_2$ $\Rightarrow$ planes are not parallel

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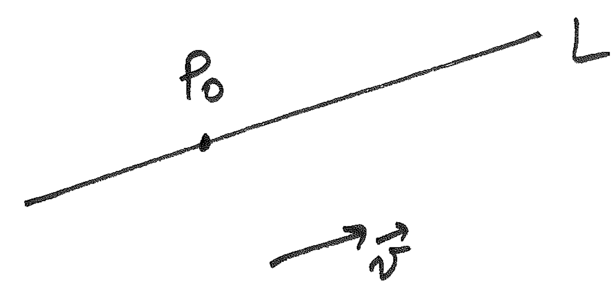
$\vec{n}_1 \cdot \vec{n}_2 = 1 \cdot 0 + 0 \cdot 1 + (-1) \cdot 2 = -2 \neq 0 \Rightarrow \vec{n}_1 \not\perp \vec{n}_2 \Rightarrow$ planes are not orthogonal

$\Rightarrow$ planes intersect at some angle θ:

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| \cdot |\vec{n}_2|} = \dots$$

Planes intersect in a line L. L belongs to both planes $\Rightarrow$ $L \perp \vec{n}_1$, $L \perp \vec{n}_2$. Hence, direction vector $\vec{v}$ which is $\parallel L$ should also: $\vec{v} \perp \vec{n}_1$, $\vec{v} \perp \vec{n}_2$. We can take

$$\vec{v} = \vec{n}_1 \perp \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{vmatrix} = \hat{i} \cdot 1 - 2\hat{j} + \hat{k} = \langle 1, -2, 1 \rangle$$



We need a pt P_0 on line L. This point $P_0(x, y, z)$ should satisfy both equations:

$$\begin{cases} x - z = 1 \\ y + 2z = 3 \end{cases}$$

2 eq^{ns}, 3 unknowns $\Rightarrow$ one unknown is a free parameter

Let $z = 0 \Rightarrow \begin{array}{l} x = 1 \\ y = 3 \end{array} \Rightarrow P_0(1, 3, 0)$

Parametric eq^{ns} of line: $\qquad v = \langle \underset{a}{1}, \underset{b}{-2}, \underset{c}{1} \rangle$

$$x = \underbrace{x_0}_{=1} + at, \quad y = \underbrace{y_0}_{=3} + bt, \quad z = \underbrace{z_0}_{=0} + ct$$

$$\therefore \boxed{x = 1 + t, \quad y = 3 - 2t, \quad z = 0 + t, \quad t \in \mathbb{R}}$$

Symmetric equations:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} \Rightarrow \boxed{\frac{x-1}{1} = \frac{y-3}{-2} = \frac{z}{1}}$$

Ex if $\vec{v} = \langle 1, 0, 2 \rangle$ $\quad P(1,3,0)$

$$x = 1 + t, \quad y = 3 + 0.t = 3, \quad z = 0 + 2t$$

$$\Downarrow$$

$$y = 3$$

$$\boxed{\frac{x-1}{1} = \frac{z-0}{2}, \quad y = 3}$$

Ex Find a vector function of a curve of intersection of cylinder $x^2+y^2=1$ and plane $y+z=2$.

Projection of cylinder $x^2+y^2=1$ on xy-plane is the circle

$$x^2+y^2=1$$

which we can parametrize as

$$x = \cos t, \quad y = \sin t$$

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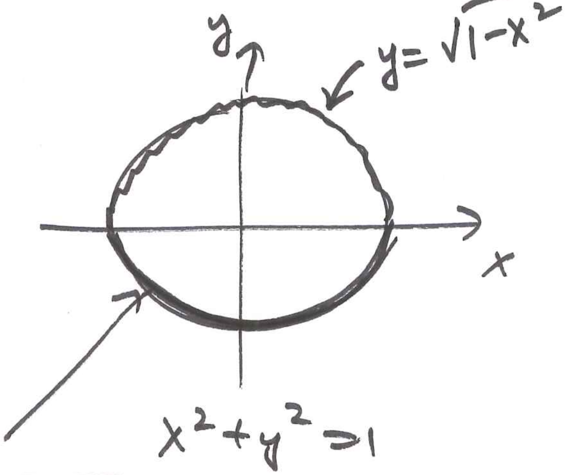
Plane: $y+z=2 \Rightarrow z=2-y=2-\sin t$

$\Rightarrow \vec{r}(t)=\cos t\,\hat{\imath}+\sin t\cdot\hat{\jmath}+(2-\sin t)\,\vec{k}$

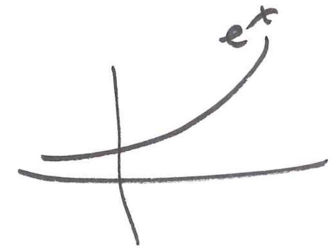


#31
S13.3

at what pt does the curve have max value?

$y=e^x$

$$K(x)=\frac{|f''(x)|}{\left[1+(f'(x))^2\right]^{3/2}}$$



$f(x)=e^x, \quad f'=f''=e^x$

$$K(x)=\frac{e^x}{(1+e^{2x})^{3/2}}$$

$= (1+e^{2x})\cdot(1+e^{2x})^{1/2}$

$$K'(x)=\frac{e^x\cdot(1+e^{2x})^{3/2}-\frac{3}{2}(1+e^{2x})^{1/2}\cdot e^{2x}\cdot 2\cdot e^x}{(1+e^{2x})^3}$$

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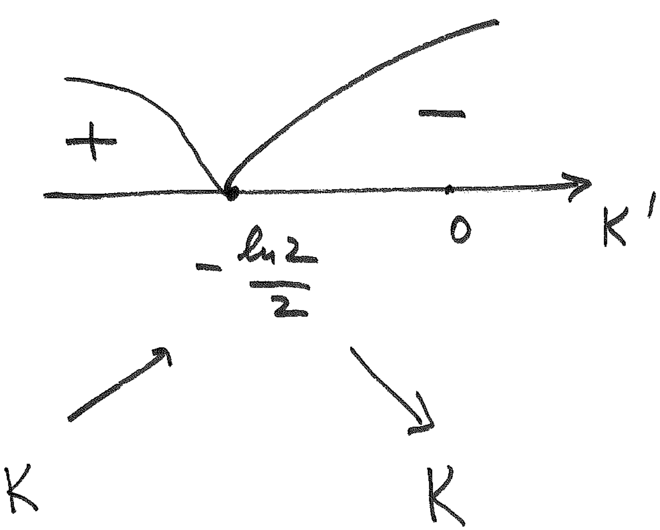
$$K'(x) = \frac{\overset{>0}{e^x}\,(1+\overset{>0}{e^{2x}})^{1/2}\cdot\left[\,1+e^{2x}-3e^{2x}\,\right]}{\underbrace{(1+e^{2x})^3}_{>0}}$$

$K'(x)=0$ to find crit. pt

$$1+e^{2x}-3e^{2x}=0$$
$$1-2e^{2x}=0$$

$$\Rightarrow [\ldots]=0 \quad \Rightarrow \quad 1-2e^{2x}=0 \quad \Rightarrow e^{2x}=\tfrac{1}{2}$$

$$2x = \ln\tfrac{1}{2} = -\ln 2 \quad \Rightarrow \quad x = -\frac{\ln 2}{2}$$



$\Rightarrow x = -\frac{\ln 2}{2}$ is a pt of max of $K(x)$

$$\therefore\ K_{max} = K\left(-\frac{\ln 2}{2}\right) = \ldots$$