Lecture 15

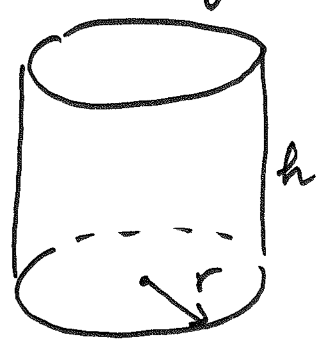
14.1 Functions of Several Variables

Ex $T = f(x,y)$: temperature T on the surface of earth

x: longitude

y: latitude

Ex



$$V = \pi r^2 \cdot h = f(r, h)$$

Def A function f of two variables is a rule that assigns to each ordered pair of real numbers x and y from domain D a unique real number denoted by $f(x,y)$.

Domain D : set of all admissible (x,y)

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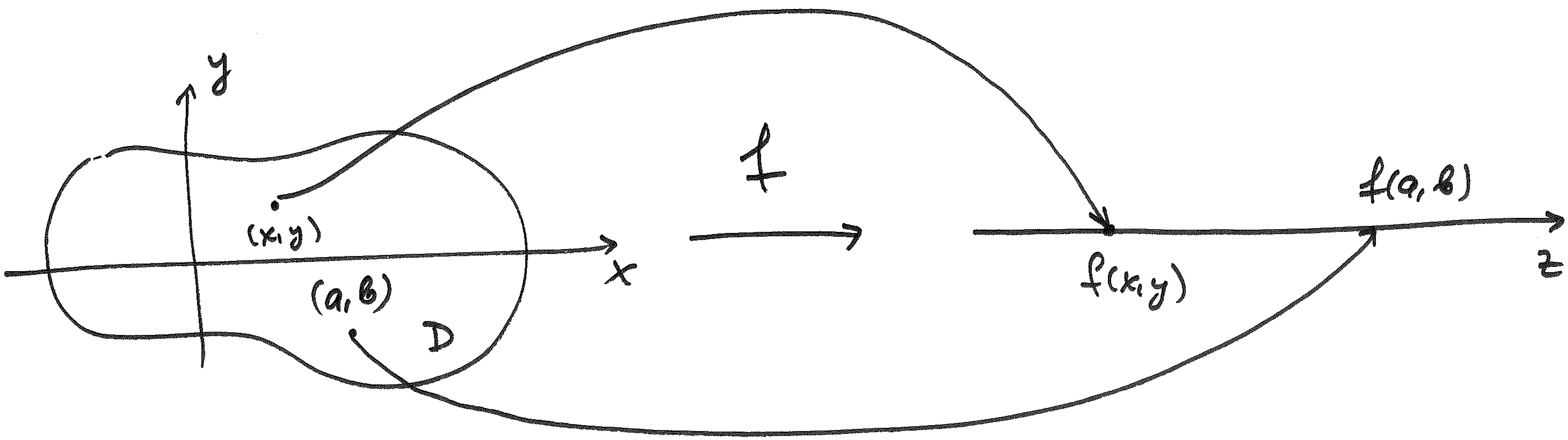
Range of f: set of all possible values of f where $(x,y) \in D$



Notation: $z = f(x,y)$

x, y: independent variables

z: dependent variable

Ex Find and sketch the domain and range of function

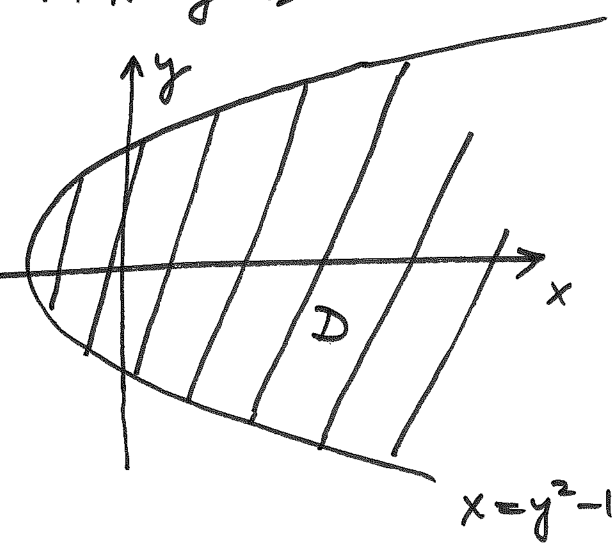
$$f(x,y) = \sqrt{1 + x - y^2}$$

Solution

f is defined for all (x,y): $1+x-y^2 \geq 0$

$x \geq y^2-1$

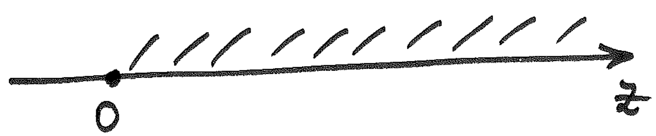
Domain $D = \{(x,y):\ x \geq y^2-1\}$



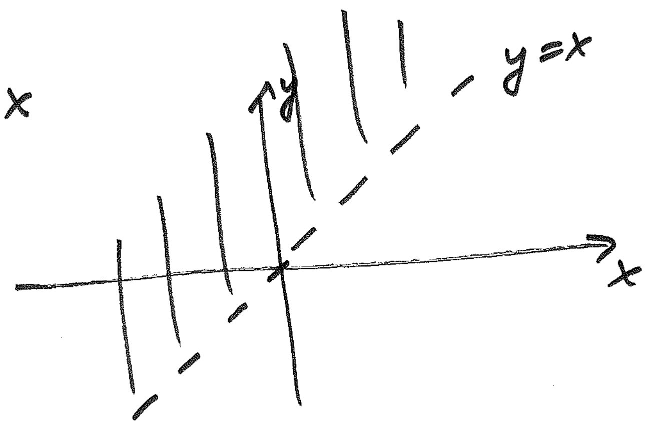
Note: boundary of D is included
$\Rightarrow$ we use a solid line

$z = f(x,y) = \sqrt{1+x-y^2} \geq 0$

$\Rightarrow z \geq 0$: range



(aside)
Ex $\quad y > x$



boundary is not included (strict inequality) $\Rightarrow$ we use dash line to sketch boundary

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Def If $f(x,y): D \to \mathbb{R}$ (f is defined for all (x,y) from D with values from $\mathbb{R}$), then its <u>graph</u> is a set of points (x,y,z) for which $z = f(x,y)$, $(x,y) \in D$.

The graph of $z = f(x,y)$ is a <u>surface.</u>

Ex $f(x,y) = ax + by + c$: linear $f^{\underline{n}}$ of x and y

$z = ax + by + c$ or $ax + by - z + c = 0$

This is an equation of a plane (recall, plane is $ax + by + cz + d = 0$)

Ex Sketch the graph of

$f(x,y) = 6 - 3x + y$: plane $\Rightarrow z = 6 - 3x + y$

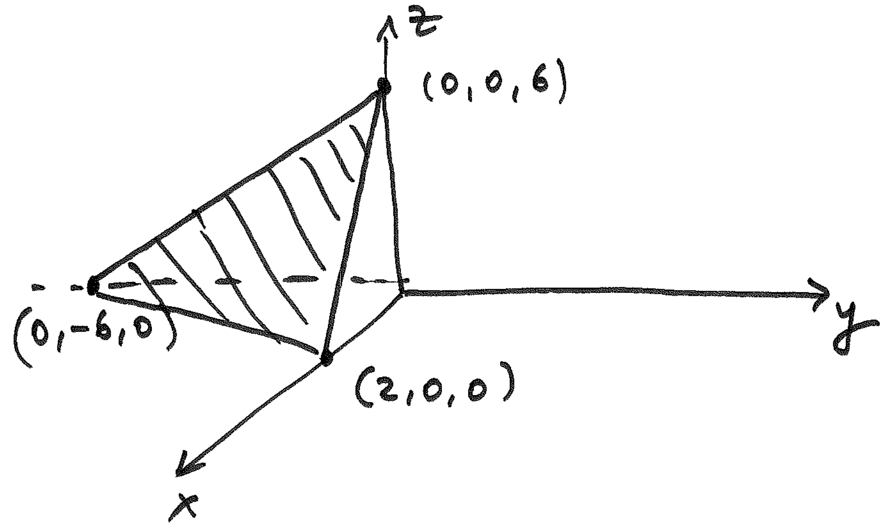
We can sketch a plane by indicating its x-, y- and z- intercepts.

$z = 6 - 3x + y$ or $-3x + y - z + 6 = 0$

$y = z = 0 \Rightarrow x = 2 \Rightarrow (2, 0, 0)$: x-intercept

$x = z = 0 \Rightarrow y = -6 \Rightarrow (0, -6, 0)$: y-intercept

$x = y = 0 \Rightarrow z = 6 \Rightarrow (0, 0, 6)$: z-intercept

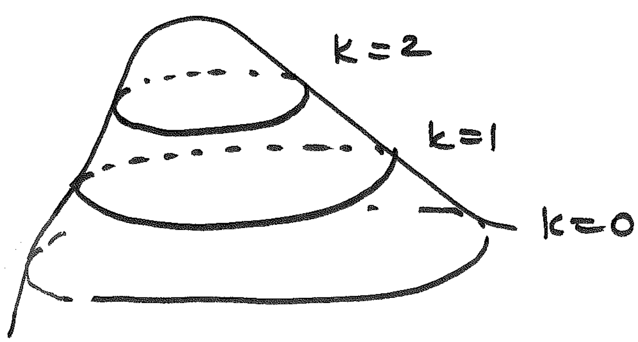


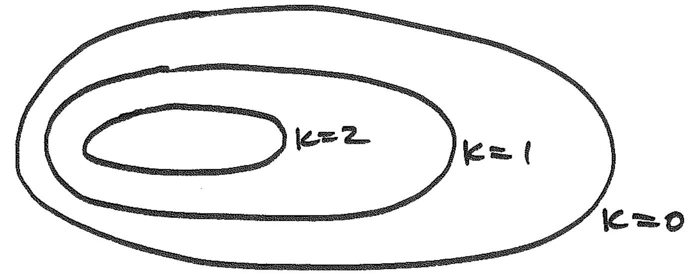
Level curves

Another method to visualize functions is to draw a contour map on which points of the same elevation (i.e. $f(x,y) = k = \text{const}$) are joined to form contour curves or level curves.

Def Level curves of function $z = f(x,y)$ are curves defined by equation $f(x,y) = k = \text{const.}$ (k is in range of f).

Level curves are traces of $f(x,y)$ with horizontal planes $z = k$.





level curves

Note In those regions where function changes quickly, the level curves are closer. In those regions where function changes slowly, level curves are farther apart.

Applications:

- topographical maps

- isothermals : curves that connect points w/ the same temperature

- isobars : the same pressure

- isoclines : the same density

<u>Ex</u> Sketch the level curves of the function

$$f(x,y) = -\sqrt{25-x^2-y^2} \leq 0$$

for $k = 0, -1, -2, -3, -4, -5$.

surface $z = f(x,y) = -\sqrt{25-x^2-y^2}$

level curves : $f(x,y) = k$ or $-\sqrt{25-x^2-y^2} = k$

$$25 - x^2 - y^2 = k^2 \qquad (-1)^2$$

or

$x^2 + y^2 = 25 - k^2$: circle centered at $(0,0)$ w/ rad. $\sqrt{25-k^2}$, $25 - k^2 \geq 0 \Rightarrow |k| \leq 5$

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$k=0$: $x^2+y^2=25=5^2$

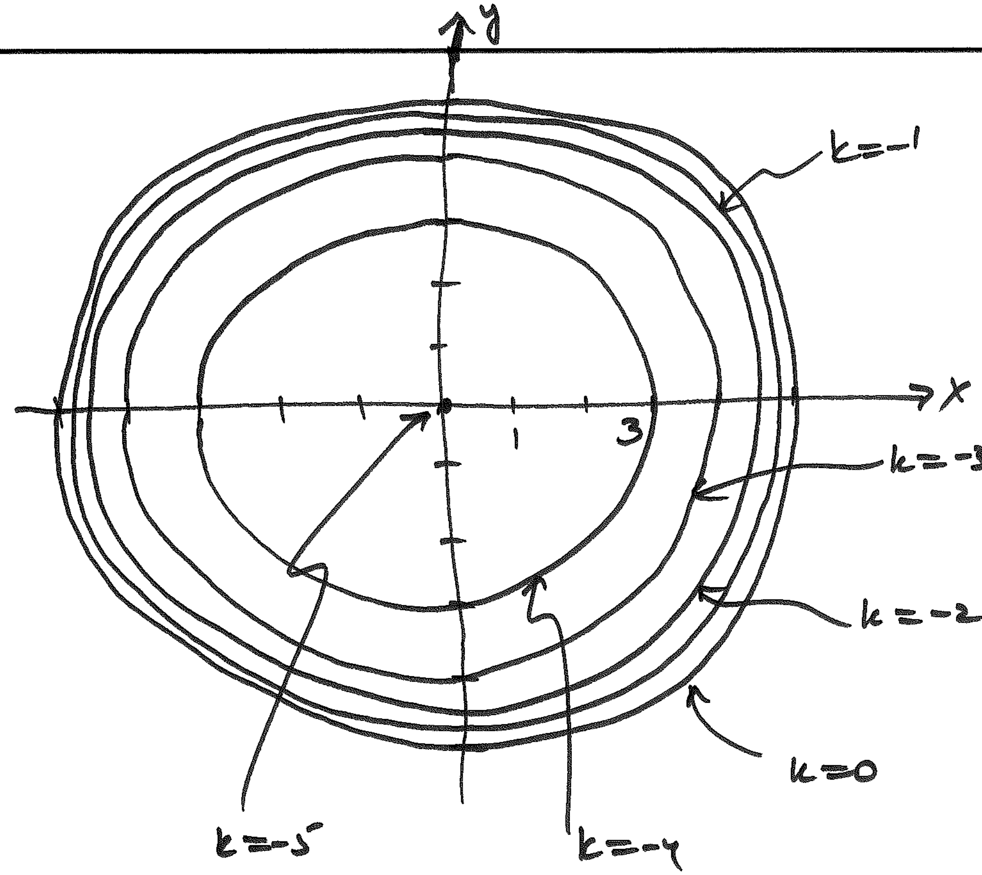
$k=-1$: $x^2+y^2=25-1=24\approx(4.899)^2$

$k=-2$: $x^2+y^2=25-4=21\approx(4.5826)^2$

$k=-3$: $x^2+y^2=25-9=16=4^2$

$k=-4$: $x^2+y^2=25-16=9=3^2$

$k=-5$: $x^2+y^2=25-25=0$



surface is hemisphere w/ $z\le 0$.

Note Functions w/ 3 or more variables are defined in a similar way.

Def a function of 3 variables, say, f, is a rule that assigns to every ordered triple (x,y,z) from domain D a unique number denoted by $f(x,y,z)$.

Ex Temperature $T=T(x,y,t)$ on surface of Earth as a function of position (x,y) and time t.