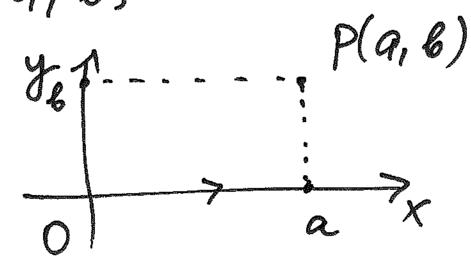
Lecture 1

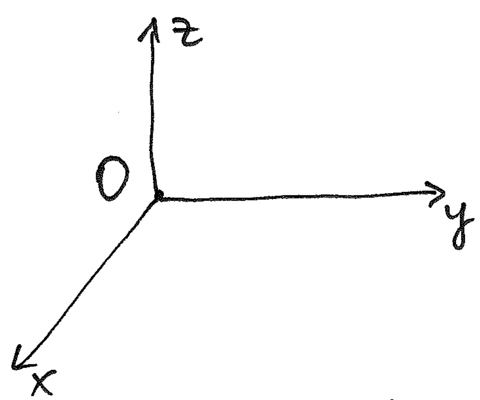
12.1 Three-dimensional coordinate systems

In 2D, a point P has two coordinates P(a, b)

(a, b): ordered pair



In 3D, P(a, b, c) ordered triple (a, b, c).

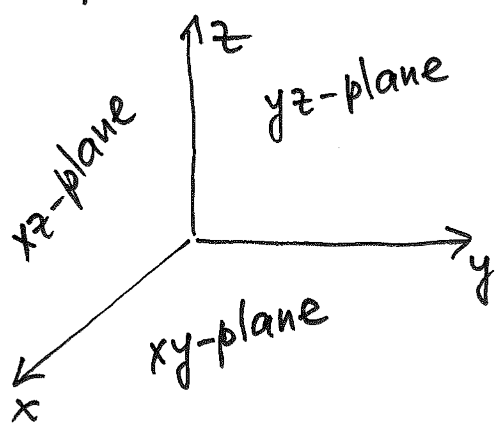


O: origin

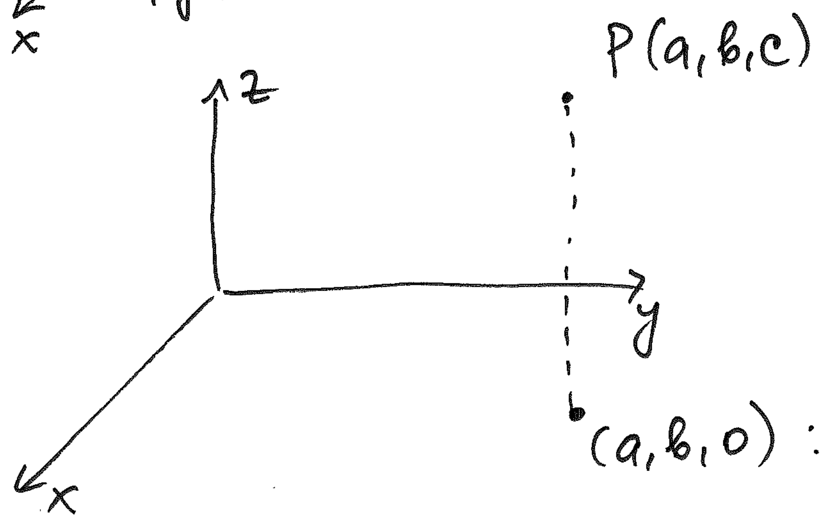
x-, y-, z-axes are mutually orthogonal and cross at O

We use right-hand rule to determine orientation of x-, y-, z-axes: you curve your fingers around z-axis, rotate from positive x- to positive y-axis in counterclockwise direction, then thumb will point in positive z-direction.

<u>or</u> thumb points in positive x-direction, index finger points in positive y-direction $\Rightarrow$ then the middle finger points in positive z-direction.

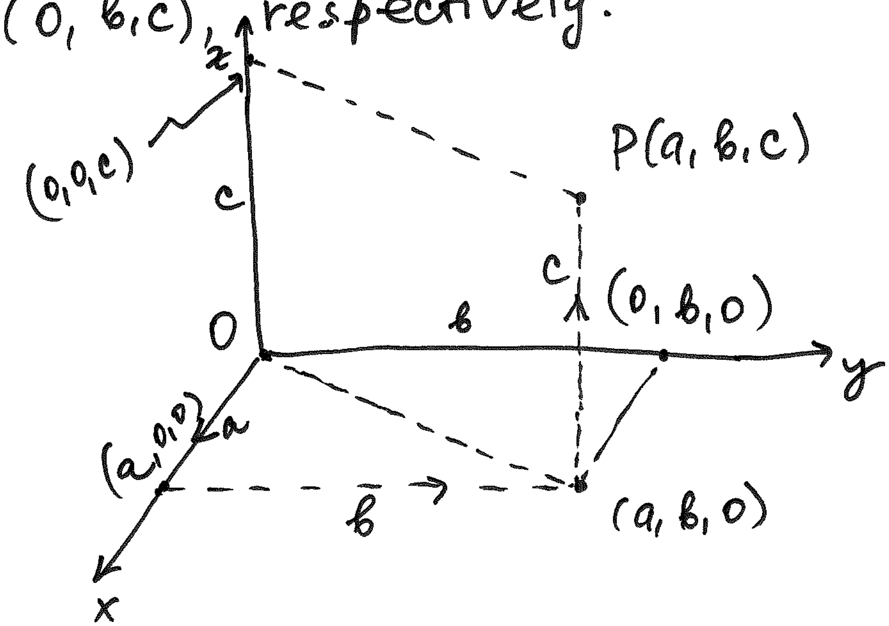


axes divide space in octants.
The principal octant is formed by positive x-, y-, z-axes (analogue of 1st quadrant): $x>0$, $y>0$, $z>0$.



$(a,b,0)$: projection of $P(a,b,c)$ onto xy-plane

Projections of P onto xz- and yz-planes are $(a,0,c)$ and $(0,b,c)$, respectively.



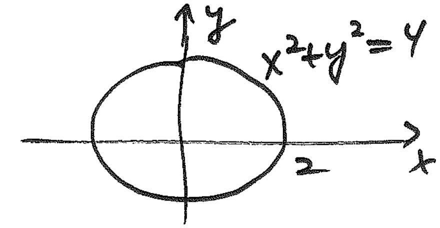
$$\mathbb{R}^3 = \mathbb{R}\times\mathbb{R}\times\mathbb{R} = \{\underset{\text{ordered triples}}{(x,y,z)} : \underbrace{x,y,z \text{ are real}}_{x,y,z\in\mathbb{R}}\}$$

belongs

There is one-to-one correspondence between set of all points in space and their coordinates in $\mathbb{R}^3$. It is called <u>three-dimensional coordinate system.</u>

Recall,

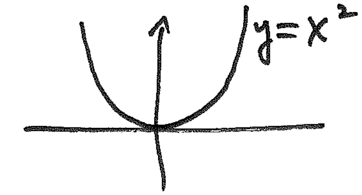
in 2D, graph of $f(x,y)=0$



Ex $x^2+y^2-4=0$

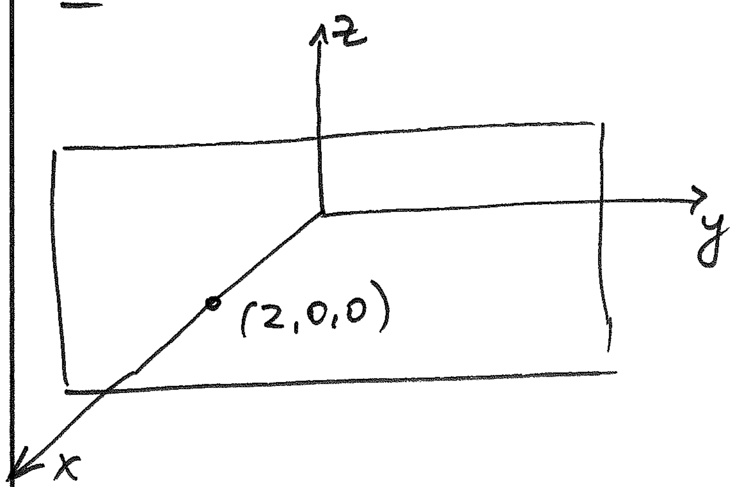
$x^2+y^2=2^2$: circle centered at origin w/ rad. $r=2$

Ex $x^2-y=0$ or $y=x^2$: parabola



In 3D, graph of $f(x,y,z)=0$ is a surface.

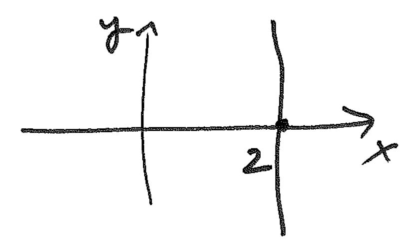
Ex which surface in $\mathbb{R}^3$ is represented by $x=2$?



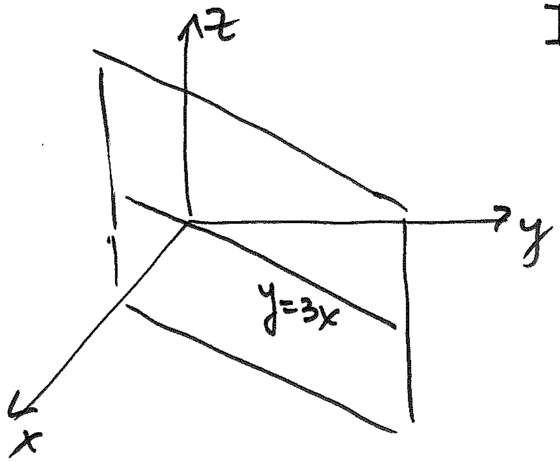
$x=2$ is a plane that goes through pt $(2,0,0)$ parallel to yz-plane

Aside

In 2D, $x=2$



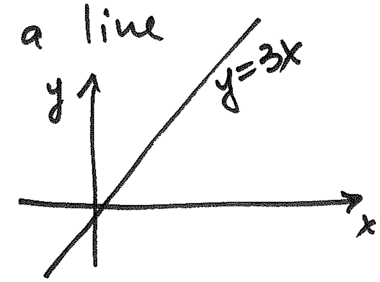
Ex Sketch surface in $\mathbb{R}^3$ given by $y = 3x$.



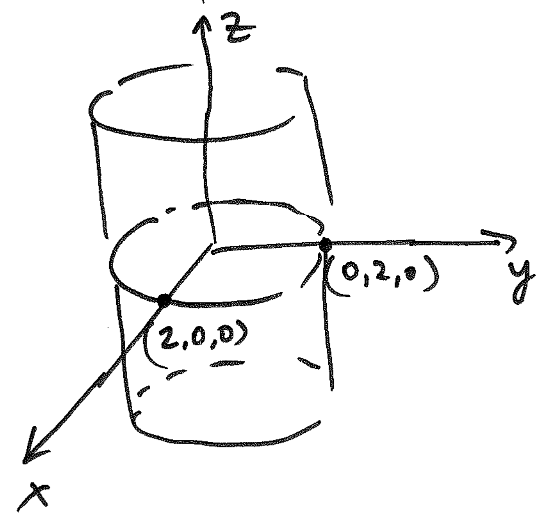
It is a vertical plane that crosses xy-plane at line $y = 3x$

Aside

In 2D, $y = 3x$ is a line



Ex $x^2 + y^2 = 4$ is a cylinder in 3D



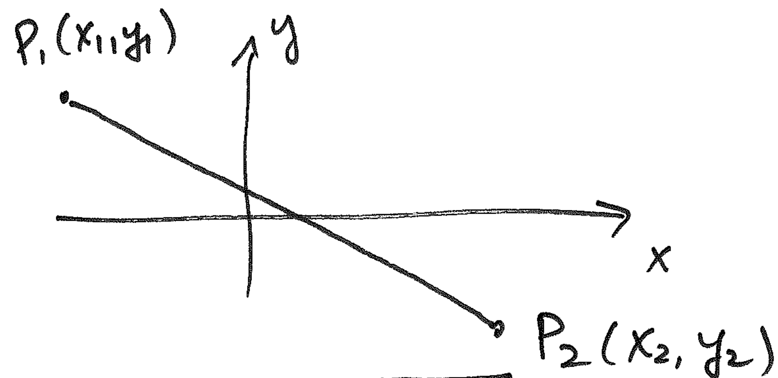
Ex $x = 0$ is yz-plane

$y = 0$ is xz-plane

$z = 0$ is xy-plane

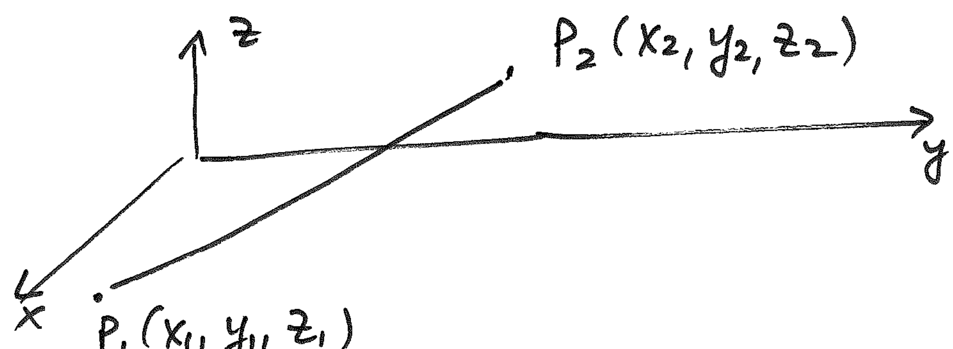
Distance Formula in 3D

In 2D



$|P_1P_2| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$: distance between P_1 and P_2

In 3D :

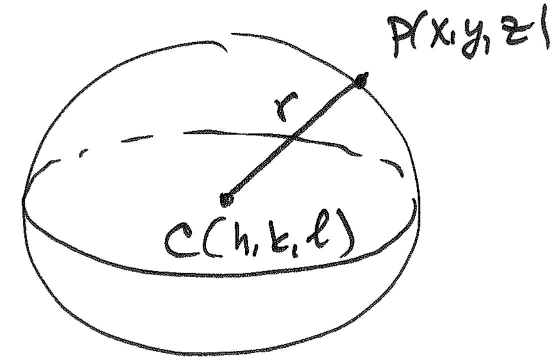


$|P_1P_2| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$: distance between P_1 and P_2 in 3D

Ex Compute distance between $P(-1,2,3)$ and $Q(0,4,-7)$.

$$|PQ| = \sqrt{(0-(-1))^2 + (4-2)^2 + (-7-3)^2} = \sqrt{105} \approx 10.25$$

Ex Find equation of a sphere with radius r and center $C(h,k,\ell)$.



a sphere is a set of points $P(x,y,z)$ that are located within distance r from center $C(h,k,\ell)$.

$$|CP| = r \quad \text{or} \quad |CP|^2 = r^2$$

$$|CP| = \sqrt{(x-h)^2 + (y-k)^2 + (z-\ell)^2}$$

or $\boxed{(x-h)^2 + (y-k)^2 + (z-\ell)^2 = r^2}$: sphere at $C(h,k,\ell)$ and rad. r

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$x^2+y^2+z^2=r^2$: sphere w/ rad r centered at origin

Ex Show that eq^n represents a sphere and find its radius and center.

$$x^2+y^2+z^2-6x+4y-2z=11$$

Idea: complete square

$$x^2-6x+y^2+4y+z^2-2z=11$$

$$(x^2-6x+9)-9+(y^2+4y+4)-4+(z^2-2z+1)-1=11$$

$$(x-3)^2+(y+2)^2+(z-1)^2=\underbrace{11+9+4+1}_{25}$$

$$\boxed{(x-3)^2+(y+2)^2+(z-1)^2=5^2}$$

sphere centered at $(3,-2,1)$ w/ rad 5