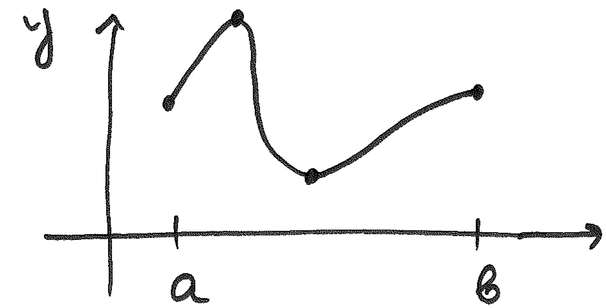
Lecture 26

Absolute Max and Absolute Min Values

Recall

$y = f(x)$, $a \le x \le b$

closed interval $[a, b]$

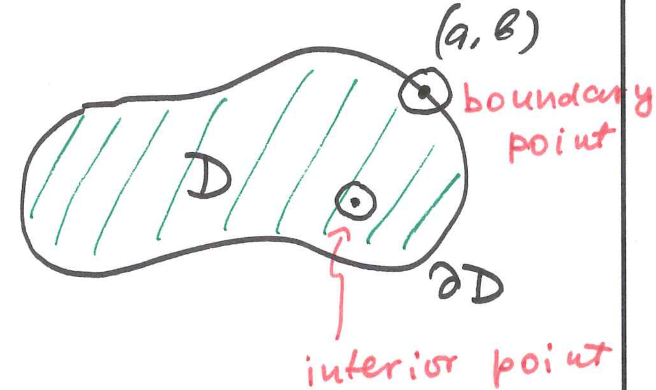


Extreme Value Thm

A continuous function $y = f(x)$ defined on a closed interval $[a, b]$ attains its ~~abs~~ max and abs min values on $[a, b]$.

To find abs. max and abs. min: we evaluate f at critical points as well as at endpoints (boundary points). The largest value is abs. max and the smallest – abs. min.

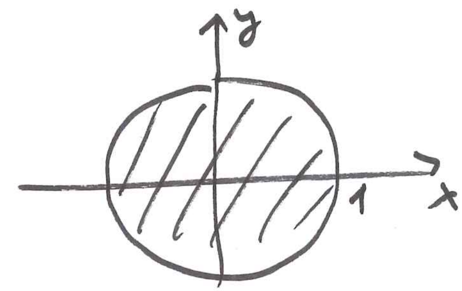
<u>Def</u> a <u>closed set</u> in $\mathbb{R}^2$ is the set that contains all boundary points.



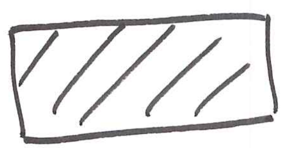
<u>Def</u> a <u>boundary point</u> (a, b) of D is a point such that every disk centered at (a, b) contains points from D and points not from D.

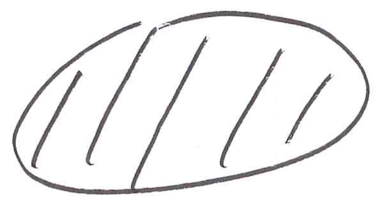
<u>Def</u> An <u>interior point</u> can be enclosed in a disk that lies completely within D.

<u>Ex</u> $D = \{(x,y) : x^2 + y^2 \leq 1\}$: closed set



<u>Ex</u>





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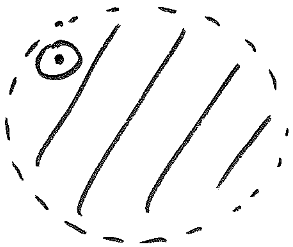
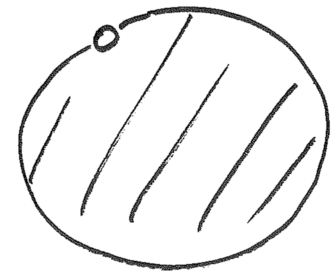


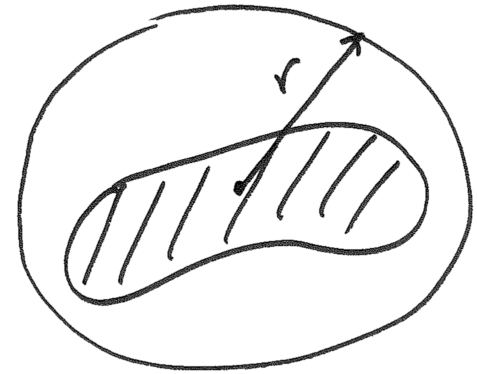
not closed
(it is <u>open set</u>:
all points are interior

not closed



not closed

Def a <u>bounded set</u> in $\mathbb{R}^2$ is a set that is contained in some disk of finite radius, i.e. set of finite extent



r: finite

<u>Extreme Value Thm for functions of two variables</u>

If function $f(x,y)$ is continuous on a closed bounded set D in $\mathbb{R}^2$, then it attains abs. max $f(x_1, y_1)$ and abs. min $f(x_2, y_2)$ values where $(x_1,y_1), (x_2,y_2) \in D$, i.e. abs max and abs. min are attained in D.

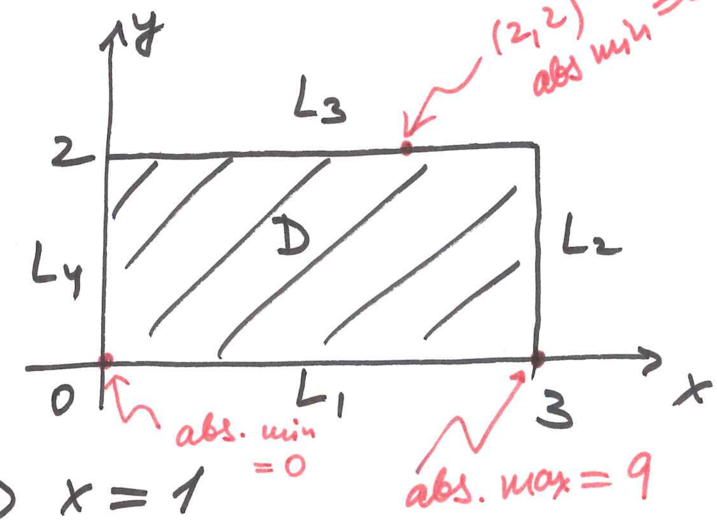
To find abs. max and abs. min values of a continuous function f:

1. Find the values of f at crit. points
2. Find extreme values of f on the boundary ∂D of D.
3. The largest is abs max, the smallest is abs min.

<u>Ex</u> Find the abs max and abs min of

$f(x,y) = x^2 - 2xy + 2y$ on the rectangle

$D = \{(x,y):\ 0 \le x \le 3,\quad 0 \le y \le 2\}$



$f_x = 2x - 2y = 2(x-y)$

$f_y = -2x + 2 = 2(1-x)$

$f_x = f_y = 0 \Rightarrow x - y = 0,\ 1 - x = 0 \Rightarrow x = 1$
$y = x = 1$

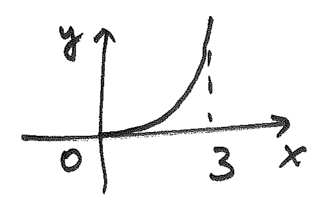
$\therefore$ (1,1) is a crit. point

Second derivative test:

$$D\big|_{(1,1)} = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}_{(1,1)} = \begin{vmatrix} 2 & -2 \\ -2 & 0 \end{vmatrix} = -4 < 0$$

$D < 0 \Rightarrow$ (1,1) is not an extremum point
(1,1) is a saddle point

f is a polynomial $\Rightarrow$ f is a continuous function defined on a closed and bounded set $\Rightarrow$ f attains its abs min and abs max values on D. Since there is no extremum inside D, f attains abs. max and abs min on the boundary ∂D of D.



L_1: $y=0$, $0 \le x \le 3 \Rightarrow f = x^2 - 2x \cdot 0 + 2 \cdot 0 = x^2$

$f_{min} = x^2\big|_{x=0} = \boxed{0}$ $\qquad f_{max} = x^2\big|_{x=3} = \boxed{9}$