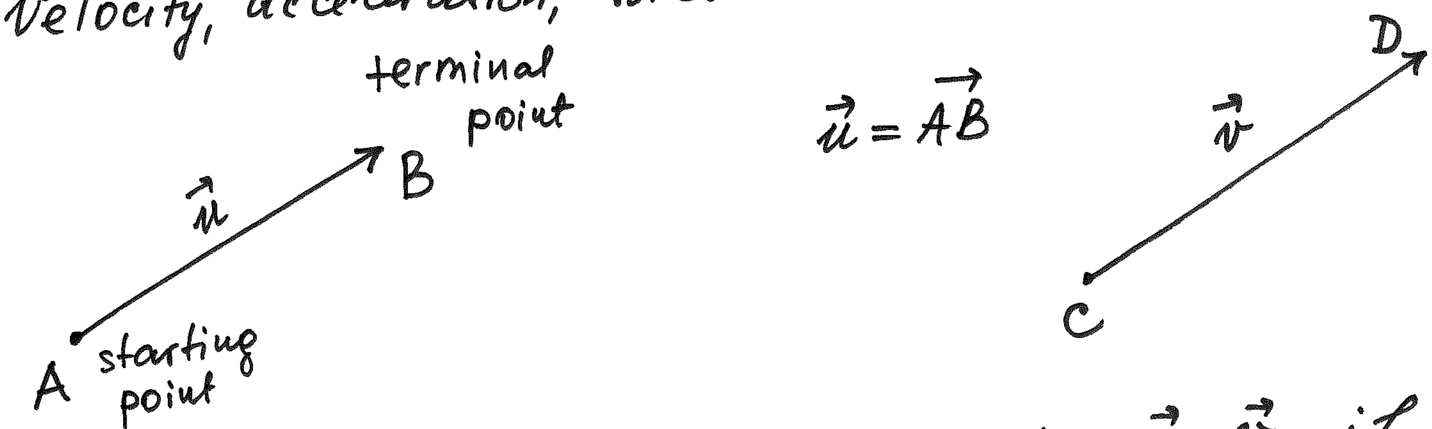
Lecture 2

12.2 Vectors

Def a vector is an object that has magnitude (length) and direction.

Ex Velocity, acceleration, force etc.

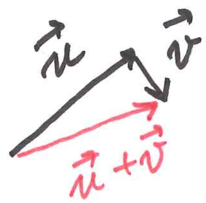


$\vec{u} = \overrightarrow{AB}$

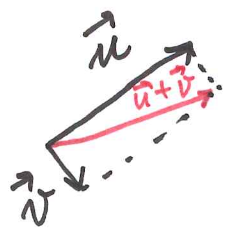
Vectors $\vec{u}$ and $\vec{v}$ are the same, i.e. $\vec{u} = \vec{v}$, if they have the same magnitude and direction.

Combining Vectors

Addition: $\vec{u} + \vec{v}$

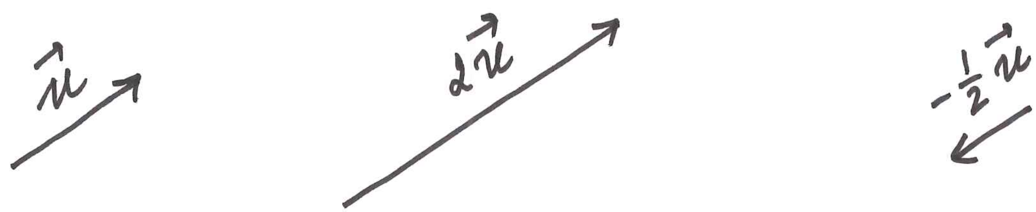


Triangle Law



Parallelogram Law

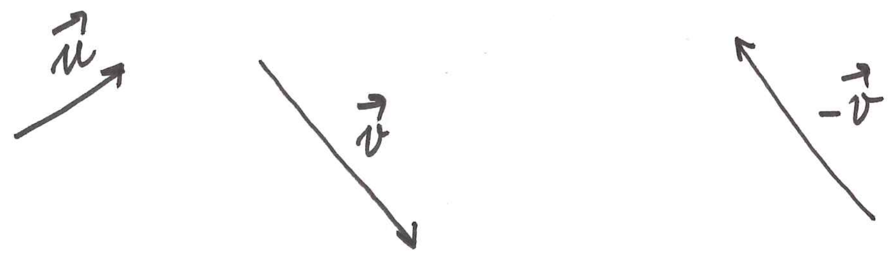
Def Let $\vec{u}$ be a vector, c be a real number. Then $c\vec{u}$ is a vector whose length is $|c|$ times length of $\vec{u}$ and direction is the same as of $\vec{u}$ if $c>0$ and opposite if $c<0$.

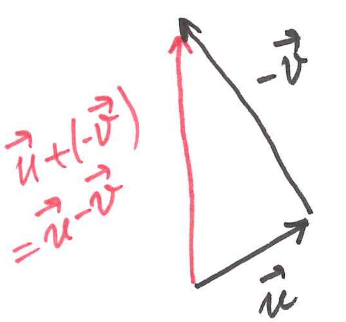


If $c=0$ or $\vec{u}=\vec{0}$ $\Rightarrow$ $c\cdot\vec{u}=\vec{0}$

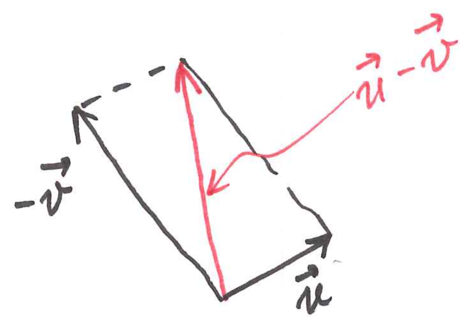
$\vec{0}$: zero vector has length 0 but no direction

<u>Difference:</u> $\vec{u}-\vec{v} = \vec{u}+(-1)\cdot\vec{v}$



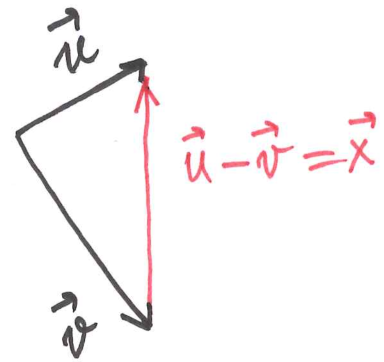


Triangle Law

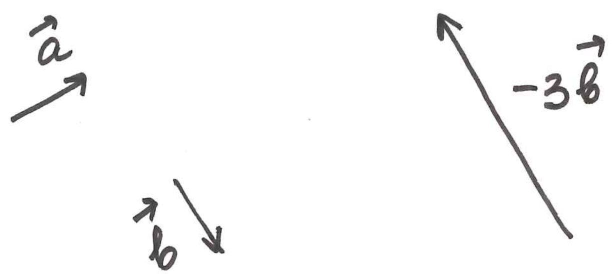


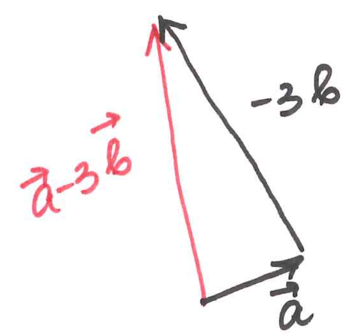
Parallelogram Law

<u>Note</u> $\vec{u}-\vec{v}=\vec{x} \Rightarrow \vec{v}+\vec{x}=\vec{u}$

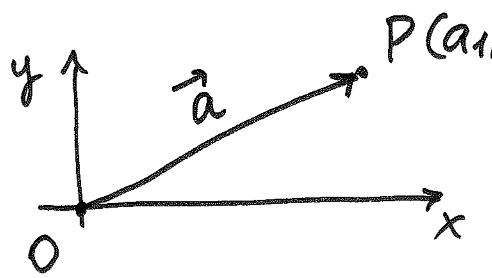


<u>Ex</u> Draw $\vec{a}-3\vec{b} = \vec{a}+(-3)\vec{b}$





Components



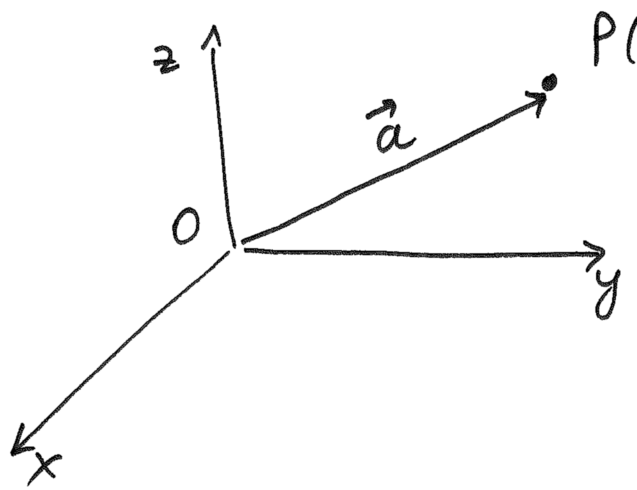
$\vec{a} = \vec{OP} = \langle a_1, a_2 \rangle$

Vector $\vec{a}$ starts at the origin and terminates at pt $P(a_1, a_2)$.

We say that $\vec{a}$ has <u>components</u> a_1 and a_2 and write

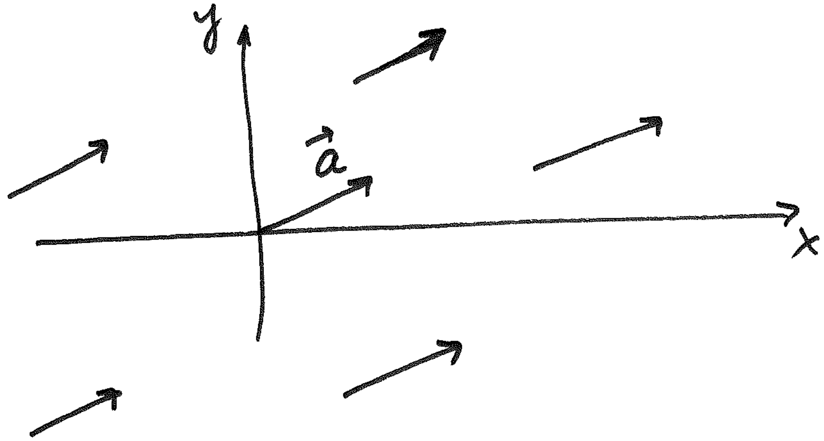
$$\vec{a} = \langle a_1, a_2 \rangle$$

3D



$\vec{a} = \langle a_1, a_2, a_3 \rangle$

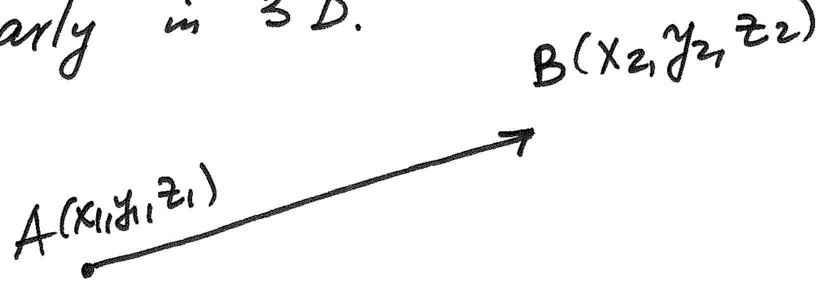
components of $\vec{a}$



All these vectors are <u>representations</u> of $\vec{a}$. They have the same length and the same direction as $\vec{a}$.

Vector $\vec{a}$ that starts at the origin is called a <u>position vector</u>.

Similarly in 3D.



We define $\vec{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

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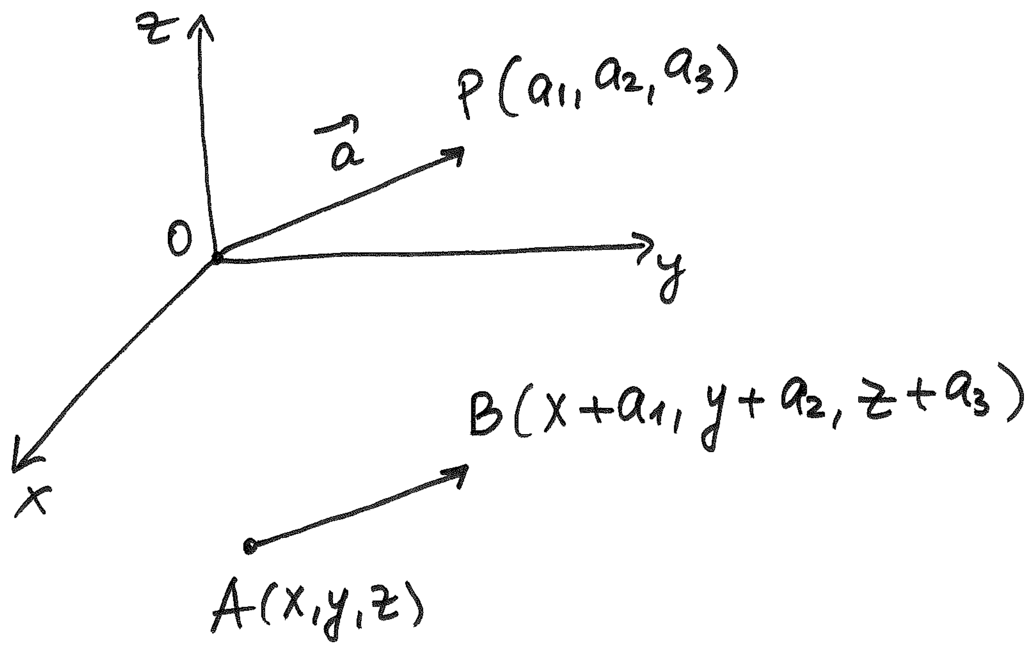
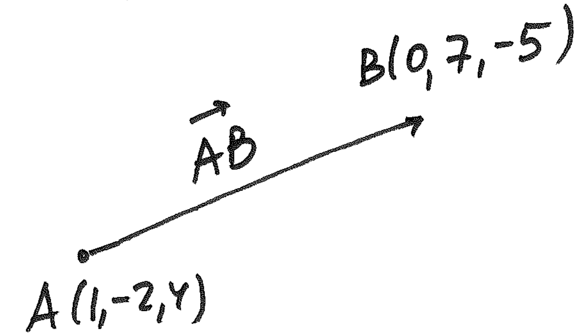


$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$

$$\vec{AB} = \vec{a}$$

Ex Find the vector represented by the directed line segment with initial point $A(1, -2, 4)$ and terminal point $B(0, 7, -5)$.

$$\vec{AB} = \langle 0-1,\ 7-(-2),\ -5-4 \rangle = \langle -1, 9, -9 \rangle$$



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Def length or magnitude of vector $\vec{a} = \langle a_1, a_2, a_3 \rangle$ is

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_2^2}$$

or $\|\vec{a}\|$

Properties of vectors

Let $\vec{a}, \vec{b}, \vec{c}$ be vectors and α, β scalars. Then

1. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
2. $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$
3. $\vec{a} + \vec{0} = \vec{a}$
4. $\vec{a} + (-\vec{a}) = \vec{0}$
5. $\alpha(\vec{a} + \vec{b}) = \alpha\vec{a} + \alpha\vec{b}$
6. $(\alpha + \beta)\vec{a} = \alpha\vec{a} + \beta\vec{a}$

7. $\alpha(\beta\vec{a}) = (\alpha\beta)\vec{a}$

8. $1 \cdot \vec{a} = \vec{a}$

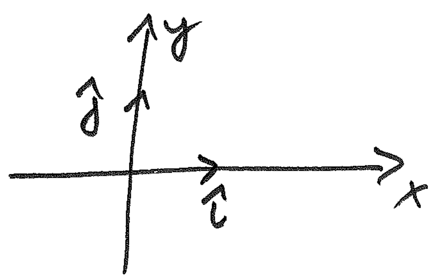
Componentwise addition and multiplication by a constant:

$\vec{a} = \langle a_1, a_2, a_3 \rangle$, $\vec{b} = \langle b_1, b_2, b_3 \rangle$ α: scalar

$\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$

$\alpha\vec{a} = \langle \alpha a_1, \alpha a_2, \alpha a_3 \rangle$

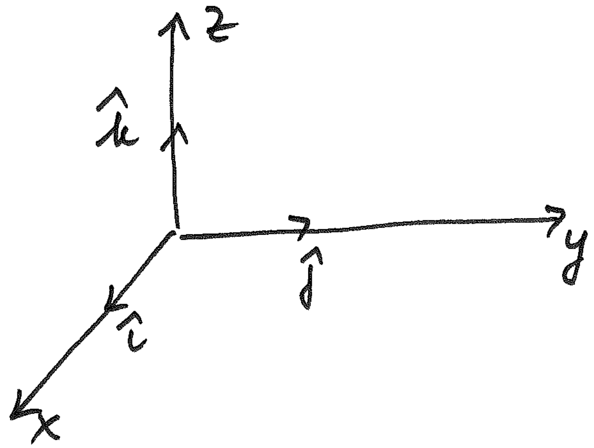
Def Unit vector is a vector whose length equals 1.



$\hat{i} = \langle 1, 0 \rangle$ special vectors

$\hat{j} = \langle 0, 1 \rangle$

$\hat{i}, \hat{j}$ are orthogonal and are unit vectors



$\hat{i} = \langle 1, 0, 0 \rangle$

$\hat{j} = \langle 0, 1, 0 \rangle$

$\hat{k} = \langle 0, 0, 1 \rangle$

mutually orthogonal special unit vectors

Given a nonzero vector $\vec{a}$, we can get a unit vector $\vec{u}$ that has the same direction as $\vec{a}$:

$$\vec{u} = \frac{1}{|\vec{a}|}\vec{a}$$

Note $\vec{a} = \langle a_1, a_2, a_3 \rangle = \langle a_1, 0, 0 \rangle + \langle 0, a_2, 0 \rangle + \langle 0, 0, a_3 \rangle =$

$= a_1\langle 1, 0, 0 \rangle + a_2\langle 0, 1, 0 \rangle + a_3\langle 0, 0, 1 \rangle = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

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Ex Given $\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} + 4\vec{k}$, find $2\vec{a} - 3\vec{b}$.

$$2\vec{a} - 3\vec{b} = 2(2\vec{i} - 3\vec{j} + \vec{k}) - 3(\vec{i} + 2\vec{j} + 4\vec{k}) =$$

$$= \underline{4\vec{i}} - \underline{6\vec{j}} + 2\vec{k} - \underline{3\vec{i}} - \underline{6\vec{j}} - 12\vec{k} = \vec{i} - 12\vec{j} - 10\vec{k}$$

or

$$2\vec{a} - 3\vec{b} = 2\langle 2, -3, 1\rangle - 3\langle 1, 2, 4\rangle = \langle 4, -6, 2\rangle$$

$$- \langle 3, 6, 12\rangle = \langle 4-3, -6-6, 2-12\rangle = \langle 1, -12, -10\rangle = \vec{i} - 12\vec{j} - 10\vec{k}$$

Note $\vec{a} = \langle a_1, a_2, a_3\rangle \Rightarrow |\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

$$\vec{u} = \frac{1}{|\vec{a}|}\vec{a} = \frac{1}{|\vec{a}|}\langle a_1, a_2, a_3\rangle = \left\langle \frac{a_1}{|\vec{a}|}, \frac{a_2}{|\vec{a}|}, \frac{a_3}{|\vec{a}|} \right\rangle$$

unit vector in the direction of $\vec{a}$