Lecture 7

Ex Find eq^{n} of line of intersection (see previous example/ Lecture 6 from 1/26/2017).

Let L be the line of intersection.

L belongs to both planes $\Rightarrow L \perp \vec{n}_1, \quad L \perp \vec{n}_2$

Let $\vec{v}$ be a direction vector of line L

$\Rightarrow \vec{v} \parallel L$ and $\vec{v} \perp \vec{n}_1, \vec{v} \perp \vec{n}_2 \Rightarrow \vec{v} \parallel (\vec{n}_1 \times n_2)$

Take $\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 4 \\ 1 & 6 & 5 \end{vmatrix} = -39\hat{i} - 6\hat{j} + 15\hat{k}$

To define line L, we need direction vector $\vec{v} = \langle -39, -6, 15 \rangle$ and some pt P_0 on the line. This point P_0 should satisfy both equations of planes (since $P_0 \in$ both planes)

$$\begin{cases} 2x-3y+4z=5 \\ x+6y+5z=3 \end{cases}$$

2 equations for 3 unknowns x, y, z

$\Rightarrow$ one variable, say x, is a free parameter, i.e. x can be any number.

Let $\boxed{x=0}$ arbitrary choice

<u>Note</u>: another choice is, say, $x=1$ etc. You may also choose another variable, say $y=0$ or $y=2$ etc.

$$\Rightarrow \begin{cases} -3y+4z=5 \\ 6y+5z=3 \end{cases}$$

Solve for y and z:

$$\Rightarrow \boxed{y=-\tfrac{1}{3}, \quad z=1}$$

$\Rightarrow P_0(0, -\frac{1}{3}, 1)$ is a pt on line L.

Now we can write equations of line L

$$x = \underset{\substack{\| \\ 0}}{x_0} + \underset{-39}{a}t, \quad y = \underset{\substack{\| \\ -\frac{1}{3}}}{y_0} + \underset{-6}{b}t, \quad z = \underset{\substack{\| \\ 1}}{z_0} + \underset{15}{c}t, \qquad t \in \mathbb{R}$$

$\vec{v} = \langle -39, -6, 15 \rangle$

$$\Rightarrow \quad x = 0 - 39t, \quad y = -\frac{1}{3} - 6t, \quad z = 1 + 15t, \qquad t \in \mathbb{R}$$

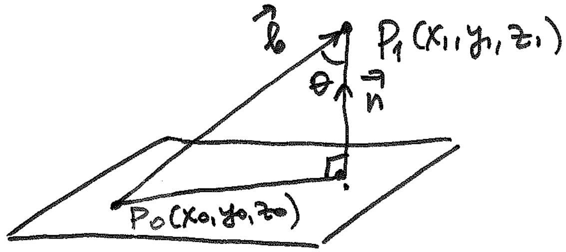
<u>parametric</u> eq^{ns} of line L of intersection

or

$$\frac{x-0}{-39} = \frac{y+\frac{1}{3}}{-6} = \frac{z-1}{15} :$$

<u>symmetric</u> eq^{ns} of line L

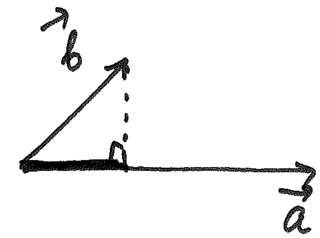
Ex Distance D from a pt $P_1(x_1, y_1, z_1)$ to the plane $ax+by+cz+d=0$



$P_0(x_0, y_0, z_0)$: arbitrary pt on the plane

$\vec{b} = \overrightarrow{P_0P_1} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$

Distance $D = |\underbrace{\text{comp}_{\vec{n}} \vec{b}}_{\text{scalar projection of } \vec{b} \text{ onto } \vec{n}}| = |\vec{b}| \cdot |\underbrace{\cos\theta}_{= \frac{\vec{n}\cdot\vec{b}}{|\vec{n}|\cdot|\vec{b}|}}| = \cancel{|\vec{b}|} \cdot \dfrac{|\vec{n}\cdot\vec{b}|}{|\vec{n}|\cdot\cancel{|\vec{b}|}} = \dfrac{|\vec{n}\cdot\vec{b}|}{|\vec{n}|} =$



$\vec{n} = \langle a, b, c \rangle$

$= \dfrac{|a(x_1 - x_0) + b(y_1 - y_0) + c(z_1 - z_0)|}{\sqrt{a^2+b^2+c^2}} \boxed{=}$

$P_0 \in$ plane $\Rightarrow$ P_0 satisfies eq^n $ax + by + cz + d = 0$, ie.

$ax_0 + by_0 + cz_0 + d = 0 \Rightarrow d = -ax_0 - by_0 - cz_0$

$\boxed{=} \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}$

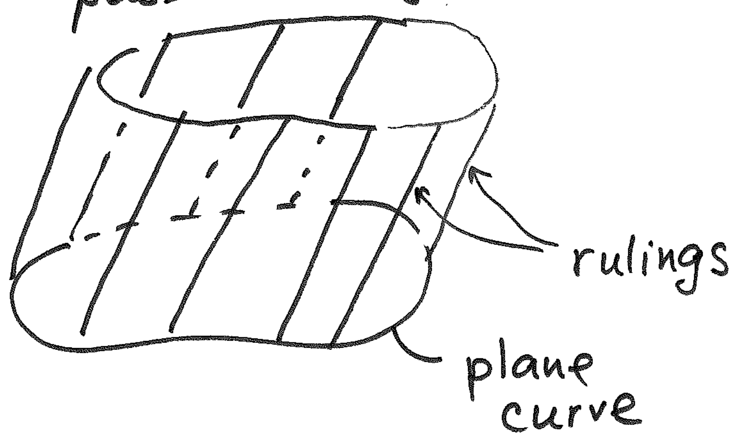
$\therefore \boxed{D = \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}}$ distance from $P_1(x_1, y_1, z_1)$ to plane $ax+by+cz+d=0$

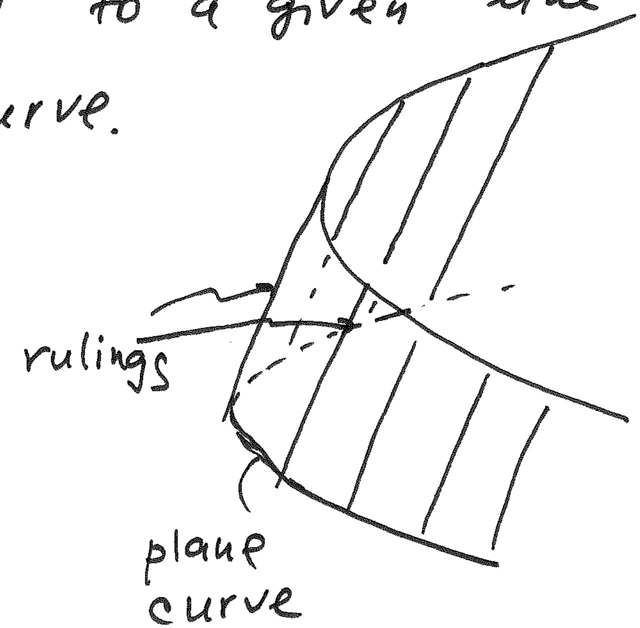
12.6 Cylinders and Quadric Surfaces

To construct / visualize surfaces, it is convenient to look at their intersections w/ coordinate planes (xy-, yz-, xz-planes). These intersections are <u>traces</u> or <u>cross-sections</u>.

Cylinders

<u>Def</u> a <u>cylinder</u> is the surface that consists of all lines (called <u>rulings</u>) that are parallel to a given line and pass through a given <u>plane</u> curve.





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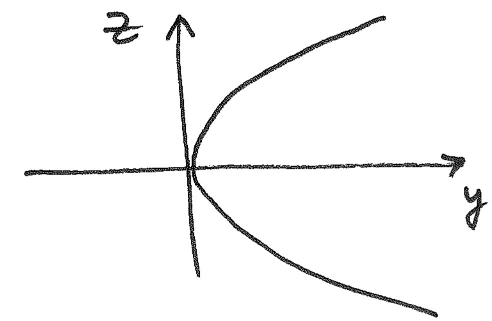
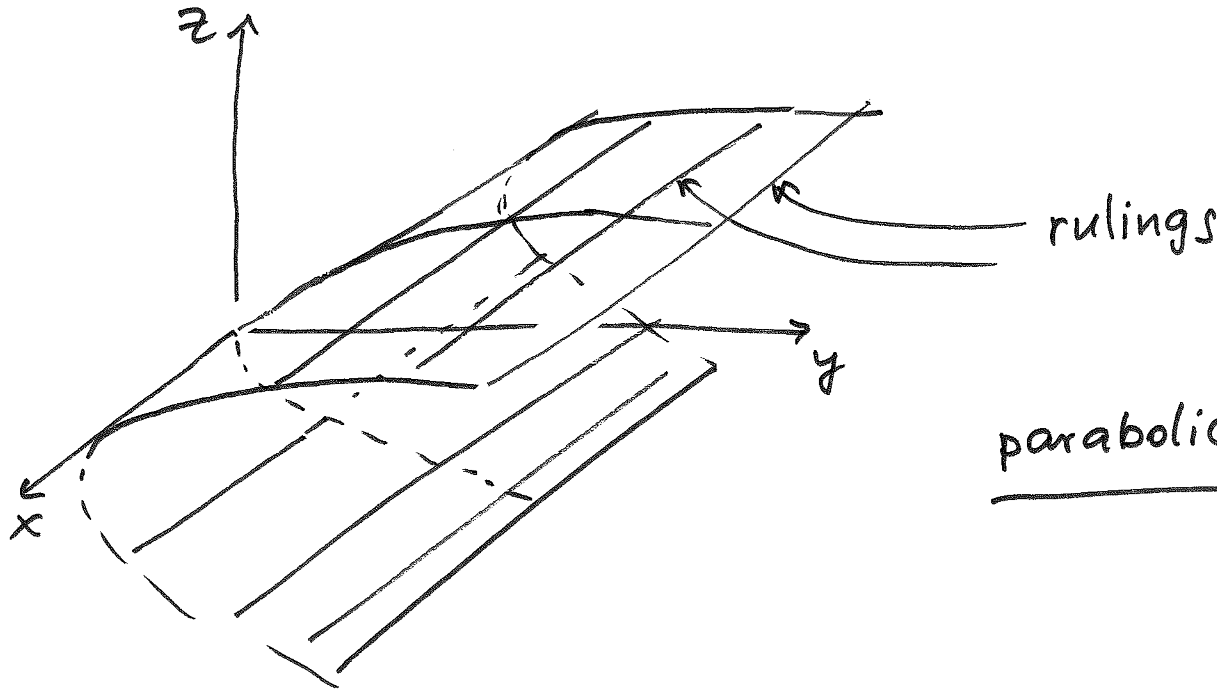
<u>Ex</u> Sketch the graph of $y = z^2$.

No x $\Rightarrow$ when $x = k$ we get the same equation $y = z^2$: parabola





<u>parabolic cylinder</u>

<u>Note</u>: rulings are $\parallel$ to x-axis (as a consequence of the fact that x is missing in $y = z^2$).

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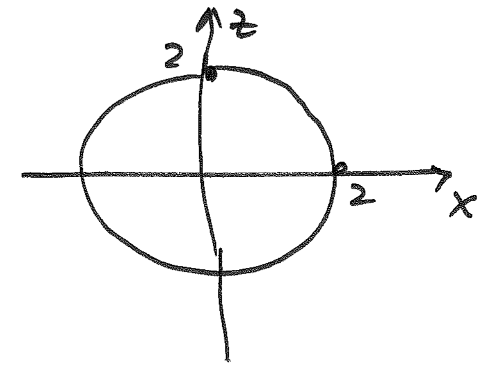
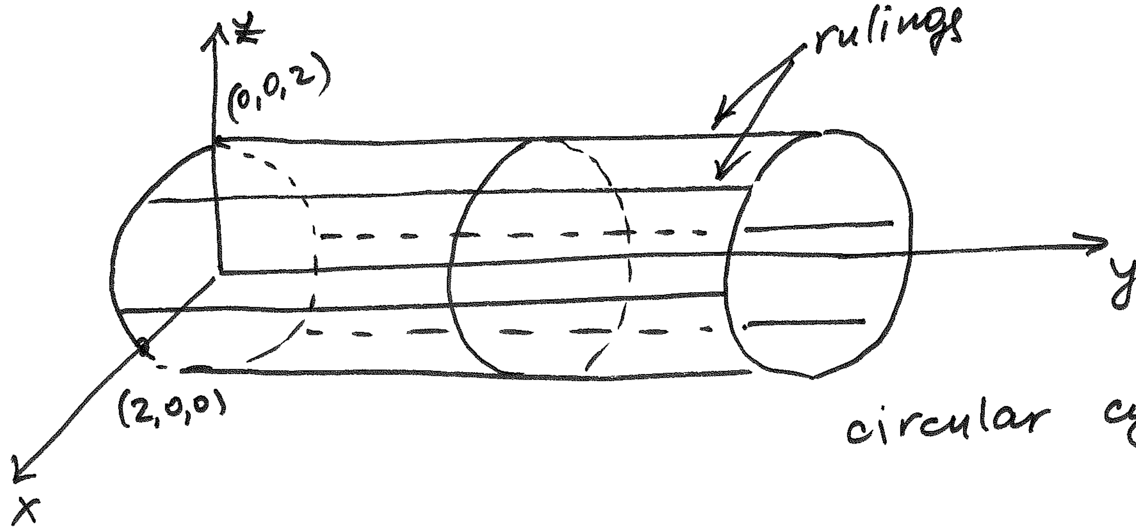
Ex Identify and sketch surface $x^2+z^2=4$.

Cross-sections w/ $y=k$ are $x^2+z^2=4$: circle centered at $(0,0)$ and radius $r=2$

plane parallel to xz-plane





circular cylinder

Note: rulings are $\parallel$ to y-axis