NAME: __________________________________________

For each problem clearly **box** your final answer. If you need additional space, continue your work on the back of the page or extra sheet at the end of the exam. **Calculators and note-cards are not allowed.**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points Possible</th>
<th>Points Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td></td>
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<tr>
<td>2</td>
<td>25</td>
<td></td>
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<td>3</td>
<td>25</td>
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<td>4</td>
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<tr>
<td>Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
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1. [25 pts]

(a) Consider a function $f : D \rightarrow \mathbb{R}$ with $D \subseteq \mathbb{R}$. State the definition of *uniform continuity* of $f$ on some set $E \subseteq D$.

(b) Show that the function

$$f(x) = \frac{1}{x^2}$$

is uniformly continuous on $x \in [1, \infty)$. 

2. [25 pts]

(a) Suppose that a function $f : D \rightarrow \mathbb{R}$ with $D \subseteq \mathbb{R}$ and $a$ is an accumulation point of $D$. State the definition of the derivative of $f$ at $x = a$.

(b) Show that the function

$$f(x) = \begin{cases} 
  x & \text{if } x \text{ is rational} \\
  0 & \text{if } x \text{ is irrational}
\end{cases}$$

is not differentiable at $x = 0$. 

3. [25 pts]

(a) State Rolle’s Theorem

(b) Prove that between two consecutive roots of \( f(x) = 1 - e^x \sin x \) there exists at least one root of \( g(x) = 1 + e^x \cos x \).

**Hint:** Note that \( 1 - e^x \sin x = 0 \) is equivalent to \( e^{-x} - \sin x = 0 \).
4. [25 pts]

(a) For a function \( f : (a, b) \rightarrow \mathbb{R} \), define \textit{concavity up}, \textit{concavity down}, and a \textit{point of inflection}.

(b) Determine where the function

\[
  f(x) = \begin{cases} 
    \frac{x^2 - 1}{x^3} & \text{if } x \neq 0 \\
    0 & \text{if } x = 0
  \end{cases}
\]

is concave up and concave down. Then find all points of inflection, if any, and sketch the graph.