MATH 471: Mid-Term Exam 3

Fall 2008
Wednesday, December 5, 2008
Prof. Lyudmyla Barannyk

NAME: ______________________________

For each problem clearly **BOX** your final answer. If you need additional space, continue your work on the back of the page or extra sheet at the end of the exam. **Calculators and note-cards are not allowed**.

Problem	Points Possible	Points Earned
1	25	
2	25	
3	25	
4	25	
Total	100	

1. [**25 pts**]

   (a) Consider a function $f : D \to \mathbb{R}$ with $D \subseteq \mathbb{R}$. State the definition of *uniform continuity* of f on some set $E \subseteq D$.

   (b) Show that the function

   $$f(x) = \frac{1}{x^2}$$

   is uniformly continuous on $x \in [1, \infty)$.

2. [**25 pts**]

   (a) Suppose that a function $f : D \to \mathbb{R}$ with $D \subseteq \mathbb{R}$ and a is an accumulation point of D. State the definition of the *derivative* of f at $x = a$.

   (b) Show that the function

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

   is not differentiable at $x = 0$.

3. [**25 pts**]

   (a) State Rolle's Theorem

   (b) Prove that between two consecutive roots of $f(x) = 1 - \mathrm{e}^x \sin x$ there exists at least one root of $g(x) = 1 + \mathrm{e}^x \cos x$.

   Hint: Note that $1 - \mathrm{e}^x \sin x = 0$ is equivalent to $\mathrm{e}^{-x} - \sin x = 0$.

4. [**25 pts**]

   (a) For a function $f : (a, b) \to \mathbb{R}$, define *concavity up*, *concavity down*, and a *point of inflection*.

   (b) Determine where the function

$$f(x) = \begin{cases} \dfrac{x^2 - 1}{x^3} & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0 \end{cases}$$

   is concave up and concave down. Then find all points of inflection, if any, and sketch the graph.