MATH 471: Final Exam

FALL 2008
Due on Tuesday, December 16 by 2:30 pm
Prof. Lyudmyla Barannyk

NAME: _____

For each problem clearly ⊠**BOX** your final answer. If you need additional space, continue your work on the back of the page or extra sheet at the end of the exam.

Problem	Points Possible	Points Earned
1	40	
2	30	
3	30	
Total	100	

1. [**40 pts**] Show that $f(x) = \dfrac{1}{x}$ is uniformly differentiable on (a, ∞) for any

   $a > 0$. Is f uniformly differentiable on $(0, \infty)$? Explain.

2. [**30 pts**]

(a) Attempt to evaluate $\lim\limits_{x \to \infty} \dfrac{\sqrt{x^2 + 1}}{x}$ using l'Hôpital's Rule directly.

(b) Evaluate $\lim\limits_{x \to \infty} \dfrac{\sqrt{x^2 + 1}}{x}$.

(c) Evaluate $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2 + 1}}{x}$.

3. [**30 pts**] Consider functions f and g, defined by

$$f(x) = \begin{cases} x + 2 & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0, \end{cases} \qquad \text{and} \qquad g(x) = \begin{cases} x^2 + 2x + 1 & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0. \end{cases}$$

Find $\displaystyle\lim_{x \to 0} \frac{f'(x)}{g'(x)}$ and $\displaystyle\lim_{x \to 0} \frac{f(x)}{g(x)}$. Does it contradict l'Hôpital's Rule?