1/12/2018

Look at the balance of energy in the control volume D.

Region D is arbitrary (arbitrary control volume). If the integrand is a continuous function of $\vec{x}$, eg. each term in (1.5) is continuous and the integral always zero for arbitrary D, then the integrand has to be zero.

Then from (1.5) $\Rightarrow$

$$\frac{\partial}{\partial t}(c\rho\theta) - P - \nabla\cdot(k\nabla\theta) = 0 \qquad (1.6)$$

DIFFERENTIAL FORM OF CONSERVATION LAW

These are good approx $\rightarrow$ If c, ρ and k are each constant (in x and t)

$$\frac{1}{c\rho} \Big/ \quad c\rho\frac{\partial\theta}{\partial t} - k\nabla^2\theta = P \qquad (1.7)$$

holds for $\forall t, \forall x$

came from $\frac{dQ}{dt}$ term

diffusion of heat

rate of production of heat

SENSIBLE "HEAT"

ie. one can fell it & measure it!

__Note__: thermal conductivity k may depend on temperature

Dimensions on LHS must be the same.

in 1D

$\nabla^2\theta = \frac{\partial^2 \theta}{\partial x^2}$

$$\frac{\partial \theta}{\partial t} - \underbrace{\frac{k}{c\rho}}_{K} \nabla^2 \theta = \frac{P}{c\rho}$$

$$\frac{\cancel{[\theta]}}{[T]} = \frac{[K]\cancel{[\theta]}}{[L^2]}$$

$K = \frac{k}{c\rho}$: THERMAL DIFFUSIVITY

$$[K] = \frac{[L]^2}{[T]}$$

$\Downarrow$

Thermal diffusivity has units $\frac{[\text{length}]^2}{[\text{time}]}$

$[T]$: unit of time

$[L]$: unit of length

$$\Rightarrow [T] = \frac{[K]}{[L]^2}$$

$$\Rightarrow t^* = \frac{t}{\frac{K}{L^2}}$$: dimensionless time variable

$$x^* = \frac{x}{L}$$: — " — x variable

$$u = \frac{\theta}{\theta_0}$$: — " — temperature

Often $P=0$

These give change of variables and we can write heat eq^n in new dimensionless variables (after dropping *)

$$\frac{\partial u}{\partial t} - \nabla^2 u = 0 \qquad (1.8)$$

Heat / diffusion eq^n

An IBVP for the diffusion eq^n is <u>WELL POSED</u> problem provided that <u>initial conditions</u> and boundary conditions are given / provided as a part of the problem. A form that gives a well posed problem is this:

IC $\quad u(\vec{x},0) = f(\vec{x})$

BC $\quad \underbrace{\alpha(\vec{x})\, u + \beta(\vec{x})\, \frac{\partial u}{\partial n}}_{\text{some combination of}} = \gamma(\vec{x}) \qquad \frac{\partial u}{\partial n} = \nabla u \cdot \hat{n}$

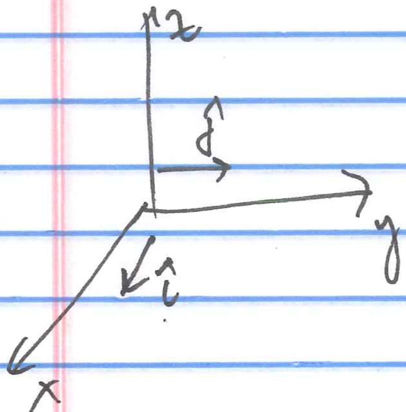
temperature and heat flux

<u>Well posed</u>: solution exists and it is unique small changes in $\alpha, \beta, \gamma \Rightarrow$ small changes in solution u, i.e. solution continuously depends on parameters.

$f(x,y)$

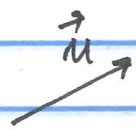
$$\lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h} = \frac{\partial f}{\partial x}$$

$$\lim_{h\to 0} \frac{f(x,y+h)-f(x,y)}{h} = \frac{\partial f}{\partial y}$$



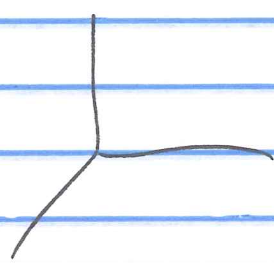
$$D_{\vec{u}} f = \lim_{h\to 0} \frac{f(x+u_1 h,\, y+u_2 h)-f(x,y)}{h}$$

$\vec{u} = (u_1, u_2)$
unit vector



$$D_{\vec{u}} f = \nabla f \cdot \vec{u}$$

unit vector



Section 1.1. Fundamental solution: Free space Green's function in 1D

Heat conduction in a metal bar. Set-up theory in 1D.

Fundamental solution of the 2nd order PDE is Green's function of that eq^{n} over an infinite domain w/ zero BC at ∞.

This is defined as the solution $F(x,t)$ of

(1.1.1) $$u_t - u_{xx} = \delta(x)\,\delta(t) \qquad x \in (-\infty,\infty),\; t>0$$

$u_t = \frac{\partial u}{\partial t}$

$u_{xx} = \frac{\partial^2 u}{\partial x^2}$

accounts for heat production inside medium at $t=0$

temperature has to tend to zero far away (unbounded domain)

$u(x,t) \to 0$ as $|x| \to \infty$

$u(x,0^-) = 0$

analogue of homog. BC

Review: Laplace transforms, contour integration (complex variables)

From (I) $\Rightarrow$

$$\frac{dQ}{dt} = \frac{d}{dt}\left[\int_D c\rho\theta\, dV\right]$$

D is not changing over time
D fixed

(IV) $$= \int_D \frac{\partial}{\partial t}(c\rho\theta)\, dV \qquad (1.4)$$

remember that full derivative becomes partial derivative, inside f^n depends on x & t.

From (II), (III), (IV) $\Rightarrow$

$$\int_D \left[\frac{\partial}{\partial t}(c\rho\theta) - P - \nabla\cdot(k\nabla\theta)\right] dV = 0 \qquad (1.5)$$

INTEGRAL FORM OF
CONSERVATION LAW