Newton's law of cooling/heating

PROBLEM

A can of coda takes 1 hr to cool from $30°C$ to $20°C$ in a refrigerator at $10°C$.

(a) Find the object's temperature 30 minutes after it started to cool.

$$y(t) = T + (y_0 - T)\,\mathrm{e}^{kt} = 10 + (30 - 10)\,\mathrm{e}^{kt} = 10 + 20\,\mathrm{e}^{kt}$$

measure t in minutes

$$y(60) = 10 + 20\,\mathrm{e}^{60k} = 20 \quad \Rightarrow \quad 20\,\mathrm{e}^{60k} = 10 \quad \Rightarrow \quad \mathrm{e}^{60k} = \frac{1}{2}$$

$$\Rightarrow \quad 60k = \ln(\frac{1}{2}) = -\ln 2 \quad \Rightarrow \quad k = -\frac{\ln 2}{60}$$

$$y(t) = 10 + 20\,\mathrm{e}^{-\frac{\ln 2}{60}t} = 10 + 20 \cdot 2^{-t/60}$$

$$y(30) = 10 + 20 \cdot 2^{-30/60} = 10 + 20 \cdot 2^{-1/2} = 10 + \frac{20}{\sqrt{2}} \sim 24°C$$

(b) How long does it take for the soda to cool an additional $6°C$ (after $y = 24°C$)?

$$y(t) = 10 + 20 \cdot 2^{-t/60} = 18$$

$$\Rightarrow \quad 20 \cdot 2^{-t/60} = 8 \quad \Rightarrow \quad 2^{-t/60} = 0.4 = \mathrm{e}^{-\frac{\ln 2}{60}t}$$

$$\Rightarrow \quad -\frac{\ln 2}{60} \cdot t = \ln 0.4 \quad \Rightarrow \quad t = 60 \cdot \frac{\ln 0.4}{\ln 0.5} \sim 80 \text{ minutes}$$

Note

As $y(t) \to T$, the rate of cooling decreases.