Math 432 - Numerical Linear Algebra - Fall 2013

<u>**Homework 10**</u>
Assigned: Sunday, November 17, 2013
Due: **Friday, November 22, 2013**

1. (a) Prove that $A^T A$ is symmetric and positive definite if and only if A has full rank.
   (b) Let x be the least-squares solution of $Ax = b$. Show that the residual $r = b - Ax$ is orthogonal to all vectors in $\mathcal{R}(A)$ in the sense that $A^T r = 0$

2. Consider the following set of data points:

x	0	1	2	3	4	5	6	7	8	9
y	1	2.9	6.8	12	20.5	30.9	42.9	51.5	73	90.5

   Using the Matlab command **vander** and the operation "\" compute the least squares fit of the data to polynomials of degrees 1 through 4.

   Plot the original data points and least-squares fits using the Matlab commands **plot** and **polyval** and compare the results. Compute the condition number of the associated Vandermonde matrix in each case.

3. Find the condition number of each of the following matrices using both generalized inverse and singular value decomposition and compare your results:

$$A = \begin{pmatrix} 0.0001 & 1 \\ 1 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 1 \\ 0.0001 & 0 \\ 0 & 0.0001 \end{pmatrix}$$

$$A = \begin{pmatrix} 7 & 6.990 \\ 4 & 4 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 0 & 7 & 8 \end{pmatrix}$$

   (Compute the generalized inverse from its definition given in Section 8.5.)

4. Let

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \\ 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix}.$$

   Find the unique least-squares solution x using
   (a) $x = A^{\dagger} b$,
   (b) the normal equations method,
   (c) the Householder QR factorization method,
   (d) the CGS and MGS methods.