Math 432 - Numerical Linear Algebra - Fall 2013

<u>Homework 11</u>
Assigned: Saturday, November 29, 2013
Due: **Friday, December 6, 2013**

Use the following data for both problems:

(i) A randomly generated matrix of order 100.

(ii) Hilbert matrix of order 20.

(iii) $\begin{pmatrix} 1 & 1 & 1 \\ \epsilon & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \epsilon \end{pmatrix}$; ϵ is such that $\mathrm{fl}(1+\epsilon^2) = 1$.

For example, $\epsilon = 10^{-8}$ would work. Why?

For each of these matrices, generate b so that the least-squares solution x in each case has all entries equal 1.

1. (*Implementation of the SVD algorithm for full-rank overdetermined least-squares problems.*) Write a Matlab program, called **lsfrsvd**, to implement Algorithm 8.4 on page 259 using reduced SVD as follows:

   $[\hat{x}] = \textbf{lsfrsvd}(A, b).$

   Test your program using the above matrices.

2. (*The purpose of this exercise is to compare the accuracy and residuals of different least-squares methods for full-rank overdetermined problems.*)

   (a) Compute the least-squares solution $\hat{x}$ for each above data set using the following:

      i. $[\hat{x}] = \textbf{lsfrmgs}(A, b)$ (least-squares using MGS).
      ii. $[\hat{x}] = \textbf{lsfrqrh}(A, b)$ (least-squares using Householder QR).
      iii. $[\hat{x}] = \textbf{lsfrnme}(A, b)$ (least-squares using normal equations).
      iv. $[\hat{x}] = \textbf{pinv}(A) * b$ (least-squares using generalized inverse).
      v. $[\hat{x}] = \textbf{lsfrsvd}(A, b)$ (least-squares using SVD).

      <u>Note</u>: **lsfrmgs**, **lsfrqrh**, and **lsfrnme** are all available in MATCOM. **pinv** is a Matlab command for computing the generalized inverse of a matrix.

   (b) Using the results of (a), make one table for each data set in the following format shown in Table 1 below. Note also that the vector x has all entries equal to 1. Write your observations.

Method	$\|\|x - \hat{x}\|\|_2 / \|\|x\|\|_2$	$\|\|A\hat{x} - b\|\|_2$
lsfrmgs		
lsfrqrh		
lsfrnme		
generalized inverse		
lsfrsvd		

Table 1: Comparison of different methods for the full-rank overdetermined least-squares problems