Assigned: January 16, 2013
Due: January 25, 2013

MATH 326: Homework 1
SPRING 2013

1. Solve the following problem using elementary calculus techniques.

$$\begin{aligned} \min \quad & (x-5)^2 + (y-5)^2 \\ \text{subject to} \quad & x + y = 5 \end{aligned}$$

[Hint: We did this in class with the goat problem. Solve for y and substitute into the objective, then use simple differentiation to find the point of optimality. Remember to show it is a minimum.]

2. Consider the following variation of Problem **??**.

$$\begin{aligned} \min \quad & (x-5)^2 + (y-5)^2 \\ \text{subject to} \quad & x + y \leq 5 \\ & x, y \geq 0 \end{aligned}$$

   (a) Draw the level sets of the objective function and the feasible region. Use this information to compute the point of optimality. [Hint: If you did Problem **??** right, you already found this point.]
   (b) Compute the gradient of the objective function at the point of optimality along with the gradients of any binding constraints at that point. [Hint: There will be only one binding constraint.]
   (c) Show that the two gradients are multiple of each other.

3. In this problem you will use elementary calculus (and a little bit of vector algebra) to show that the gradient of a simple function is always perpendicular to its level sets:

   (a) Find the gradient of $z(x, y) = x^2 + y^2$ evaluated at point (x_0, y_0).
   (b) Take any level curve set $x^2 + y^2 = k$ for $z(x, y) = x^2 + y^2$. Use implicit differentiation to find dy/dx. When you evaluate this at point (x_0, y_0), this is the slope of a tangent line to the circle $x^2 + y^2 = k$ at (x_0, y_0).
   (c) Find an expression for a vector parallel to the tangent line at (x_0, y_0). [Hint: You can use the slope you just found.]
   (d) Show that the vector you just found is perpendicular to the gradient found in the first step. [Hint: Use the dot product.]