Math 395/CS 395 - Analysis of Algorithms - Spring 2022

Homework 1

Due: **March 24, 2022**

- Include a cover page and a problem sheet.
- Read sections 1.1, 1.2, 2.1-2.3, 3.1, 3.2.

PROBLEMS:

0. Give a brief description of your academic background and research interests. If you work in a lab or research group, describe your project. Do you have a programming experience? If so, which programming language do you know? One paragraph is fine.

1. Suppose we are comparing implementations of insertion sort and merge sort on the same machine. For inputs of size n, insertion sort runs in $8n^2$ steps, while merge sort runs in $64n \lg n$ steps. For which values of n does insertion sort beat merge sort?

2. Consider the **searching problem**:

   Input: A sequence of n numbers $A = \langle a_1, a_2, \ldots, a_n \rangle$ and a value ν.

   Output: An index i such that $\nu = A[i]$ or the special value *NIL* if ν does not appear in A.

   Write pseudocode for **linear search**, which scans through the sequence, looking for ν. Using a loop invariant, prove that your algorithm is correct. Make sure that your loop invariant fulfills the three necessary properties.

3. Express the function $n^3/1000 - 100n^2 - 100n + 3$ in terms of Θ-notation.

4. Consider sorting n numbers stored in array A by first finding the smallest element of A and exchanging it with the element in $A[1]$. Then find the second smallest element of A, and exchange it with $A[2]$. Continue in this manner for the first $n - 1$ elements of A. Write pseudocode for this algorithm, which is know as **selection sort**. Which loop invariant does this algorithm maintain? Why does it need to run for only $n - 1$ elements, rather than all n elements? Give the best-case and worst-case running times of selection sort in Θ-notation.