Math 432 - Numerical Linear Algebra - Fall 2013

<u>**Homework 1**</u>

Assigned: Friday, August 30, 2013

Due: **Friday, September 6, 2013**

- Include a cover page and a problem sheet.

0. Give a brief description of your academic background and research interests. If you work in a lab or research group, give your supervisor's name and describe your project. One paragraph is fine.

1. Given matrices

$$A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 0 & 5 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 1 & 0 \\ -3 & -1 & 5 \\ 1 & 3 & 4 \end{pmatrix}, \quad C = \begin{pmatrix} 4 & 2 \\ 3 & -1 \\ 2 & -4 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & -1 & 4 \\ 0 & 2 & -2 \\ 0 & 0 & 3 \end{pmatrix}$$

compute the indicated matrices. If an operation cannot be performed, indicate why not.

(a) $2A + C^T$ and $C - 3B$

(b) CA and AC

(c) $C^T D$ and BA^T

(d) $\det(D)$ and $\det(A)$

2. Let A be a nonsingular matrix.

(a) Show that A^{-1} is unique.

(b) Show that A^{-1} is nonsingular and $(A^{-1})^{-1} = A$.

(c) Show that A^T is nonsingular and $(A^T)^{-1} = (A^{-1})^T$.

(d) If B is nonsingular, show that AB is nonsingular and $(AB)^{-1} = B^{-1}A^{-1}$.

3. Calculate the determinant of the matrix

$$\begin{pmatrix} 1 & 0 & 4 & 1 \\ -2 & 1 & -3 & 2 \\ 0 & 0 & 0 & 2 \\ 3 & 2 & 1 & -1 \end{pmatrix}$$

by first expanding along the second column.

4. Let $D = \text{diag}[d_{11}, d_{22}, \ldots, d_{nn}]$ be an $n \times n$ diagonal matrix. Show that $\det(D) = d_{11}d_{22}\ldots d_{nn}$.

5. Let α be a real number and let

$$A = \begin{pmatrix} \alpha & 4 \\ 1 & \alpha \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2 & \alpha & 0 \\ -3 & -1 & 5 \\ 1 & 3 & \alpha \end{pmatrix}.$$

   (a) For what value(s) of α is A singular?

   (b) For what value(s) of α is B singular?

6. Compute the spectrum of matrix

$$A = \begin{pmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{pmatrix}$$

   Recall that the spectrum of a matrix is the set of all its eigenvalues.