Math 571 - Functional Analysis I - Fall 2017

<u>**Homework 1**</u>

Due: **Friday, September 1, 2017**

- Include a problem sheet.

1. Show that the real line is a metric space.

2. Does $d(x, y) = (x - y)^2$ define a metric on the set of all real numbers?

3. Find all metrics on a set X consisting of two points. Consisting of one point.

4. (**Triangle inequality**) The triangle inequality has several useful consequences. For instance, using the generalized triangle inequality

$$d(x_1, x_n) \leq d(x_1, x_2) + d(x_2, x_3) + \cdots + d(x_{n-1}, x_n)$$

show that

$$|d(x, y) - d(z, w)| \leq d(x, z) + d(y, w).$$

5. Show that in sequence space s with metric

$$d(x, y) = \sum_{j=1}^{\infty} \frac{1}{2^j} \frac{|\xi_j - \eta_j|}{1 + |\xi_j - \eta_j|}$$

(see example 1.2-1 on pg. 9-11) we can obtain another metric by replacing $1/2^j$ with $\mu_j > 0$ such that $\sum \mu_j$ converges.

6. (**Space** l^p) Find a sequence which converges to 0, but is not in any space l^p, where $1 \leq p < \infty$.

7. (**Diameter, bounded set**) The *diameter* $\delta(A)$ of a nonempty set A in a metric space (X, d) is defined to be

$$\delta(A) = \sup_{x, y \in A} d(x, y).$$

A is said to be *bounded* if $\delta(A) < \infty$. Show that $A \subset B$ implies $\delta(A) \leq \delta(B)$.