Math 395/CS 395 - Analysis of Algorithms - Spring 2022

Homework 2 Due: Friday, April 1, 2022

- Include a problem sheet.
- Read sections 4.1 4.5.

PROBLEMS:

- 1. Using Figure 2.4 from the textbook by Cormen as a model, illustrate the operation of merge sort on the array $A = \langle 3, 41, 52, 26, 38, 57, 9, 49 \rangle$.
- 2. Rewrite the Merge procedure so that it does not use sentinels, instead stopping once either array L or R has had all its elements copied back to A and then copying the remainder of the other array back into A.
- 3. Use mathematical induction to show that when n is an exact power of 2, the solution of the recurrence

$$T(n) = \begin{cases} 2 & \text{if } n = 2, \\ 2T(n/2) + n & \text{if } n = 2^k, \text{ for } k > 1 \end{cases}$$

is $T(n) = n \lg n$.

- 4. We can express insertion sort as a recursive procedure as follows. In order to sort A[1..n], we recursively sort A[1..n-1] and then insert A[n] into the sorted array A[1..n-1]. Write a recurrence for the worst-case running time of this recursive version of insertion sort.
- 5. Referring back to the searching problem (see problem 2 from HW 1), observe that if the sequence A is sorted, we can check the midpoint of the sequence against ν and eliminate half of the sequence from further consideration. The **binary search** algorithm repeats this procedure, halving the size of the remaining portion of the sequence each time. Write pseudocode, either iterative or recursive, for binary search. Argue that the worst-case running time of binary search is $\Theta(\lg n)$.