Homework 2
Assigned: Friday, September 6, 2013
Due: **Friday, September 13, 2013**

- Include a cover page and a problem sheet.

1. Verify that the l_∞-norm

$$||x||_\infty = \max_{1 \leq i \leq n} |x_i|$$

   satisfies the properties of a vector norm.

2. Compute the l_1-norm, l_2-norm and l_∞-norm for each of the following vectors.

   (a) $x = (\, 3 \quad -5 \quad \sqrt{2}\,)^T$ (b) $x = (\, 2 \quad 1 \quad -3 \quad 4\,)^T$

3. Show that $||x||_\infty \leq ||x||_1 \leq n||x||_\infty$ for all $x \in \mathbb{R}^n$.

4. Compute the l_1-norm, l_2-norm and l_∞-norm for each of the following matrices.

   (a) $x = \begin{pmatrix} 5 & -4 \\ -1 & 7 \end{pmatrix}$ (b) $x = \begin{pmatrix} 4 & -1 & -2 \\ 1 & 2 & -3 \\ 0 & 0 & 4 \end{pmatrix}$

5. Show that if A is a matrix with $\rho(A) < 1$, then the matrix $I - A$ is nonsingular. (*Hint*: Assume that $I - A$ is singular and show this leads to the conclusion that $\lambda = 1$ is an eigenvalue of A.)