Math 571 - Functional Analysis I - Fall 2017

Homework 2

Due: **Monday, September 11, 2017**

1. What is an open ball $B(x_0; 1)$ on $\mathbb{R}$? In $\mathbb{C}$? In $C[a, b]$? Explain Fig. 1.

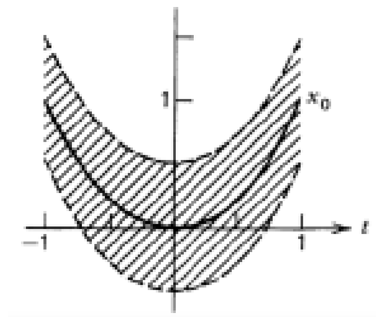

Figure 1: Region containing the graphs of all $x \in C[-1, 1]$ which constitute the ϵ-neighborhood, with $\epsilon = 1/2$, of $x_0 \in C[-1, 1]$ given by $x_0(t) = t^2$

2. If x_0 is an accumulation point of a set $A \subset (X, d)$, show that any neighborhood of x_0 contains infinitely many points of A.

3. Show that the closure $\overline{B(x_0; r)}$ of an open ball $B(x_0; r)$ in a metric space can differ from the closed ball $\tilde{B}(x_0; r)$.

4. Show that $A \subset \bar{A}$, $\bar{\bar{A}} = \bar{A}$, $\overline{A \cup B} = \bar{A} \cup \bar{B}$, $\overline{A \cap B} \subset \bar{A} \cap \bar{B}$.

5. (**Continuous mapping**) Show that a mapping $T : X \to Y$ is continuous if and only if the inverse image of any closed set $M \subset Y$ is a closed set in X.

6. (**Subsequences**) If a sequence (x_n) in a metric space X is convergent and has limit x, show that every subsequence (x_{n_k}) of (x_n) is convergent and has the same limit x.

7. If (x_n) is Cauchy and has a convergent subsequence, say, $x_{n_k} \to x$, show that (x_n) is convergent with the limit x.