MATH 326: HOMEWORK 3 SPRING 2013

1. Show that the vectors

$$x_1 = \begin{bmatrix} 1\\2\\3 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 4\\5\\6 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 7\\8\\9 \end{bmatrix}$$

are linearly dependent. [Hint: Following the example shown in class, create a matrix whose columns are the vectors in question and solve a matrix equation with right-hand-side equal to zero. Using Gauss-Jordan elimination, show that a zero row results and thus find the infinite set of values solving the system.]

- 2. Show that the vectors from Problem 1 are not a basis for \mathbb{R}^3 .
- 3. A chemical manufacturer produces three chemicals: A, B and C. These chemical are produced by two processes: 1 and 2. Running process 1 for 1 hour costs \$4 and yields 3 units of chemical A, 1 unit of chemical B and 1 unit of chemical C. Running process 2 for 1 hour costs \$1 and produces 1 units of chemical A, and 1 unit of chemical B (but no C). To meet customer demand, at least 12 units of chemical A, 5 units of chemical B and 3 units of chemical C must be produced daily. Assume that the chemical manufacturer wants to minimize the cost of production. Develop a linear programming problem describing the constraints and objectives of the chemical manufacturer and solve the problem using the graphical method. [Hint: Let x_1 be the number of times Process 1 is executed and let x_2 be the number of times Process 2 is executed. Remember, you are solving a minimization problem!]
- 4. Convert the linear problem you found in Problem 3 to standard form. Also write the standard form in its matrix formulation. In the plot you made in Problem 2, identify the value for all variables (decision and slack) at each corner on the boundary of the feasible region. [Hint: The feasible region should be unbounded, but the problem does have a bounded solution.]
- 5. Use Gauss-Jordan elimination to determine whether the following matrix has an inverse:

$$A = \left[\begin{array}{rrrr} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{array} \right]$$