Math 432 - Numerical Linear Algebra - Fall 2013

Homework 3

Assigned: Friday, September 13, 2013

Due: **Friday, September 20, 2013**

- Include a cover page and a problem sheet.

1. Let $\beta = 10$, $t = 4$. Compute $\mathrm{fl}(A^T A)$, where

$$\begin{pmatrix} 1 & 1 \\ 10^{-4} & 0 \\ 0 & 10^{-4} \end{pmatrix}.$$

   Repeat your computation with $t = 9$. Compare the results.

2. Show how to arrange computation in each of the following, so that the loss of significant digits can be avoided. Do one numerical example in each case to support your answer.

   (a) $\dfrac{1}{x} - \dfrac{1}{x+1}$ for large values of x.

   (b) $x - \sin x$ for values of x near 0.

   (c) $\dfrac{\mathrm{e}^x - 1}{x}$ for $|x| \ll 1$.

   (d) $\dfrac{1 - \cos x}{x^2}$ for small x.

3. What are the relative and absolute errors in approximating

   (a) π by $\frac{22}{7}$?

   (b) $\frac{1}{3}$ by 0.333?

   How many significant digits are there in each computation?

4. Consider evaluating

$$e = \sqrt{a^2 + b^2}.$$

   How can the computation be organized so that overflow in computing $a^2 + b^2$ for large values of a or b can be avoided?

5. Show that the integral

$$y_i = \int_0^1 \frac{x^i}{x+5} dx$$

can be computed by using the recursive formula:

$$y_i = \frac{1}{i} - 5y_{i-1}.$$

Compute $y_1, y_2, \ldots, y_{10}$ using this formula, taking

$$y_0 = \ln(x+5)|_{x=0}^{1} = \ln 6 - \ln 5 = \ln(1.2).$$

What abnormalities do you observe in this computation? Explain what happened. Now rearrange the recursion so that the values of y_i can be computed more accurately.