Math 571 - Functional Analysis I - Fall 2017

Homework 3

Due: **Wednesday, September 20, 2017**

1. (# 4, Section 1.4) Show that a Cauchy sequence is bounded.

2. (# 5, Section 1.4) Is boundedness of a sequence in a metric space sufficient for the sequence to be Cauchy? Convergent?

3. (# 5, Section 1.5) Show that the set X of all integers with metric d defined by $d(m,n) = |m-n|$ is a complete metric space.

4. (# 6, Section 1.5) Show that the set of all real numbers constitutes an incomplete metric space if we choose

$$d(x,y) = |\arctan x - \arctan y|.$$

5. (# 7, Section 1.5) Let X be the set of all positive integers and $d(m,n) = |m^{-1} - n^{-1}|$. Show that (X,d) is not complete.

6. (# 8, Section 1.5) Show that the subspace $Y \subset C[a,b]$ consisting of all $x \in C[a,b]$ such that $x(a) = x(b)$ is complete.

7. (# 13-14, Section 1.5) Show that in example 1.5-9, another Cauchy sequence is (x_n), where

$$x_n(t) = \begin{cases} n, & \text{if } 0 \le t \le n^{-2} \\ t^{-1/2} & \text{if } n^{-2} \le t \le 1. \end{cases}$$

   Show that this Cauchy sequence does not converge.