Math 432 - Numerical Linear Algebra - Fall 2013

Homework 4

Assigned: Friday, September 20, 2013

Due: **Friday, September 27, 2013**

- Include a cover page and a problem sheet.

1. (# 4.3 (c)) Show that the roots of the polynomial $(x-1)(x-0.99)(x-2)$ are ill-conditioned and give reasons for your answer.

For **Matlab resources** please see page 78 of the textbook. Matlab is available in all computer labs on campus as well as via VLAB

2. (# 4.9) Consider the matrix

$$\begin{pmatrix} 12 & 11 & 10 & \dots & 3 & 2 & 1 \\ 11 & 11 & 10 & \dots & 3 & 2 & 1 \\ 0 & 10 & 10 & \ddots & & \vdots & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \vdots & \vdots \\ \vdots & & & \ddots & \ddots & 2 & \vdots \\ 0 & \dots & \dots & \dots & 0 & 1 & 1 \end{pmatrix}$$

Find the eigenvalues of this matrix using Matlab command **eig**. Now perturb the $(1,12)$ elements by 10^{-9} and compute the eigenvalues of this perturbed matrix. What conclusion do you make about the conditioning of the eigenvalues?

3. (# 4.16 - 4.17 (a))
   (a) How are $\text{Cond}(A)$ and $\text{Cond}(A^{-1})$ related?
   (b) Show that
      i. $\text{Cond}(A) \geq 1$ for a norm $||.||$ such that $||I|| \geq 1$;
      ii. $\text{Cond}_2(A^TA) = (\text{Cond}_2(A))^2$;
      iii. $\text{Cond}(cA) = \text{Cond}(A)$ for any given norm;
   (c) Let A be an orthogonal matrix. Then show that $\text{Cond}_2(A) = 1$.

4. (# M4.2) Using the function **for**, write a Matlab program to find the *inner product* and the *outer product* of two n-vectors u and v.

$$[s] = inpro(u,v)$$

$$[A] = outpro(u,v)$$

Test your program on vectors $u = (\,1 \quad 2 \quad 3 \quad 4\,)^T$ and $v = (\,5 \quad 6 \quad 7 \quad 8\,)^T$.

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Homework 4, updated problem # 5

Due: **Monday, September 30, 2013**

5. (# M4.10)

   (a) Write a Matlab program to construct the $n \times n$ lower triangular matrix $A = (a_{ij})$ as follows:

   $$a_{ij} = 1 \quad \text{if} \quad i = j,$$
   $$a_{ij} = -1 \quad \text{if} \quad i > j,$$
   $$a_{ij} = 0 \quad \text{if} \quad i < j.$$

   (b) Perform an experiment to show that the solution of $Ax = b$ with A as above and the vector b created such that $Ax = b$, where $x = \text{rand}(n, 1)$, becomes more and more inaccurate as n increases due to increasing ill-conditioning of A. Let $\hat{x}$ denote the computed solution.

   Present your results in the following form

n	Cond(A)	Relative error $\frac{\|\|x-\hat{x}\|\|_2}{\|\|x\|\|_2}$	Residual norm $\frac{\|\|b-A\hat{x}\|\|_2}{\|\|b\|\|_2}$
10			
20			
30			
40			
50			

   Print x and $\hat{x}$ for each value of n as column vectors next to each other.

Note: Please include printouts of your programs and output results.