**Math 432 - Numerical Linear Algebra - Fall 2013**

**Homework 5**

Assigned: Saturday, September 28, 2013

Due: **Friday, October 4, 2013**

- Include a cover page and a problem sheet.
- Include all of your scripts and output results.
- Place a comment at the top of each function or script that you submit which includes the name of the function or script

1. Let $A$ be the $2 \times 2$ matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ . Use Gaussian elimination to obtain $A^{-1}$ by solving the two systems $Ax_1 = e_1$ and $Ax_2 = e_2$, where $e_1$ and $e_2$ are the columns of the $2 \times 2$ identity matrix. Note that you can perform both at the same time by considering the augmented system $[A|I]$. Show that $A^{-1}$ exists if and only if $\det(A) \neq 0$.

2. Let $M_1 = \begin{pmatrix} 1 & 0 & 0 \\ m_{2,1} & 1 & 0 \\ m_{3,1} & 0 & 1 \end{pmatrix}$ , $M_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & m_{3,2} & 1 \end{pmatrix}$ , $P_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ .

   (a) Show that
   $$M_1^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -m_{2,1} & 1 & 0 \\ -m_{3,1} & 0 & 1 \end{pmatrix} .$$

   (b) Show that
   $$M_1^{-1}M_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -m_{2,1} & 1 & 0 \\ -m_{3,1} & -m_{3,2} & 1 \end{pmatrix} .$$

   (c) Show that $P_1^{-1}=P_1$.

3. Find the $LU$ factorization of $A$ and use it to solve $Ax = b$.
$$A = \begin{pmatrix} 4 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 4 \end{pmatrix} , \ b = \begin{pmatrix} 2 \\ -3 \\ 3 \\ -2 \end{pmatrix} .$$

4. Consider the problem $Ax = b$ where $A$ is a tridiagonal matrix. What is the operation count for the forward elimination and the back substitution steps of Gaussian elimination in this case? Count add/sub and mult/div operations separately, then give the overall order of the total operations needed. (Use $O(n^p)$ notation).

5. **Back and Forward Substitution: Matlab Program**

Write two programs, one that performs back substitution on an upper triangular matrix $U$ and another that performs forward elimination on a lower triangular matrix $L$ (you may assume that the diagonal entries of $L$ are all 1). The program for back substitution should begin with:

```
function [x] = backsub(U, b)
n=length(b);
(your code here)
```

where $U\mathbf{x} = \mathbf{b}$ and $U$ is an upper triangular matrix. Similarly, a program for forward elimination should start with

```
function [x] = forwardsub(L, b)
n=length(b);
(your code here)
```

where $L\mathbf{x} = \mathbf{b}$ and $L$ is lower triangular. Test your code on the following systems:

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -1 \\ 0 & 0 & 2 \end{bmatrix} \mathbf{y} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Remember, in Matlab you can solve matrix equations as follows (assuming you have defined the matrix $A$ and the rhs vector $\mathbf{b}$):

```
>> A\b
```

You can use Matlab solutions to check your results. Print and hand-in the text file containing your program as well as outputs.