Math 571 - Functional Analysis I - Fall 2017

Homework 5

Due: **Friday, October 6, 2017**

1. (# 2, Section 2.3) Show that c_0, the space of all sequences of scalars converging to zero (see problem # 1, Section 2.3), is a closed is closed subspace of l^∞, so that c_0 is complete by Result 1.5-2 and Theorem 1.4-7.

2. (# 3, Section 2.3) In l^∞, let Y be the subset of all sequences with only finitely many nonzero terms. Show that Y is a subspace of l^∞ but not a closed subspace.

3. (# 5, Section 2.3) Show that $x_n \to x$ and $y_n \to y$ implies $x_n + y_n \to x + y$. Show that $\alpha_n \to \alpha$ and $x_n \to x$ implies $\alpha_n x_n \to \alpha x$.

4. (# 6, Section 2.3) Show that the closure $\bar{Y}$ of a subspace Y of a normed space X is again a vector space.

5. (# 1, Section 2.4) Give examples of subspaces of l^∞ and l^2 which are not closed.

6. (# 2, Section 2.4) What is the largest possible c in (from Lemma 2.4-1 on linear combinations)

$$||\alpha_1 x_1 + \ldots + \alpha_n x_n|| \geq c(|\alpha_1| + \ldots + |\alpha_n|)$$

   if $X = \mathbb{R}^2$ and $x_1 = (1, 0)$, $x_2 = (0, 1)$? If $X = \mathbb{R}^3$ and $x_1 = (1, 0, 0)$, $x_2 = (0, 1, 0)$, $x_3 = (0, 0, 1)$?

7. (# 8, Section 2.4) Show that the norms $||.||_1$ and $||.||_2$ satisfy (see also problem # 8, Section 2.2)

$$\frac{1}{\sqrt{n}}||x||_1 \leq ||x||_2 \leq ||x||_1,$$

   where

$$||x||_1 = |\xi_1| + |\xi_2| + \ldots + |\xi_n|$$

$$||x||_2 = \left(|\xi_1|^2 + |\xi_2|^2 + \ldots + |\xi_n|^2\right)^{1/2}.$$