Math 571 - Functional Analysis I - Fall 2017

<u>**Homework 6**</u>
Due: **Friday, October 13, 2017**

1. (# 1, Section 2.5) Show that $\mathbb{R}^n$ and $\mathbb{C}^n$ are not compact.

2. (# 3, Section 2.5) Give examples of compact and noncompact curves in the plane $\mathbb{R}^2$.

3. (# 4, Section 2.5) Show that for an infinite subset M in the space s (cf. 2.2-8) to be compact, it is necessary that there are numbers $\gamma_1, \gamma_2, \ldots$ such that for all $x = (\xi_k(x)) \in M$ we have $|\xi_k(x)| \leq \gamma_k$. (It can be shown that the condition is also sufficient for the compactness of M.)

4. (# 1, Section 2.6) Show that the operator identity operator (2.6-2), the zero operator (2.6-3) and the differentiation operator (2.6-4) are linear.

5. (# 2, Section 2.6) Show that the operators $T_1, \ldots T_4$ from $\mathbb{R}^2$ to $\mathbb{R}^2$ defined by

$$(\xi_1, \xi_2) \to (\xi_1, 0)$$

$$(\xi_1, \xi_2) \to (0, \xi_2)$$

$$(\xi_1, \xi_2) \to (\xi_2, \xi_1)$$

$$(\xi_1, \xi_2) \to (\gamma\xi_1, \gamma\xi_2)$$

   respectively, are linear, and interpret these operators geometrically.

6. (# 3, Section 2.6) What are the domain, range and null space of T_1, T_2, T_3 in problem 5 (# 2, Section 2.6)?

7. (# 7, Section 2.6) Let X be any vector space and $S : X \to X$ and $T : X \to X$ any operators. S and T are said to *commute* if $ST = TS$, that is $(ST)x = (TS)x$ for all $x \in X$. Do T_1 and T_3 problem 5 (# 2, Section 2.6) commute?