Math 432 - Numerical Linear Algebra - Fall 2013

Homework 7

Assigned: Saturday, October 26, 2013

Due: **Friday, November 1, 2013**

- Include a cover page and a problem sheet.
- Include all of your scripts and output results.
- Place a comment at the top of each function or script that you submit which includes the name of the function or script

1. Consider the matrix

$$\begin{pmatrix} a & -1 & 0 \\ -1 & 4 & 1 \\ 0 & 1 & 5 \end{pmatrix}$$

   (a) For what values of a will this matrix be positive definite?

   (b) For what values of a will this matrix be strictly diagonally dominant?

2. Compute the Cholesky factorization $A = HH^T$ of

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1.001 & 1.001 \\ 1 & 1 & 2 \end{pmatrix}$$

   Solve the system $Ax = b$, where

$$b = \begin{pmatrix} 3 \\ 3.0020 \\ 4.0010 \end{pmatrix}$$

   using the obtained Cholesky factorization. Verify your answer using the MATCOM program **CHOLES** or Matlab program **chol**. Note that Matlab program **chol**(A) computes the Cholesky factor R such that $A = R^T R$, where R is upper triangular.

3. Let $H = I - \frac{2uu^T}{u^T u}$ be a Householder matrix. Then prove that

   (a) $Hu = -u$

   (b) $Hv = v$ if $v^T u = 0$.

4. Given vector $x = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}^T$, compute a Householder matrix $H = I - \frac{2uu^T}{u^T u}$ such that Hx has zeros in the positions 2 through 4. Compute Hx.

5. Find QR factorization of

$$A = \begin{pmatrix} 10 & 1 & 1 & 1 \\ 2 & 10 & 1 & 1 \\ 1 & 1 & 10 & 1 \\ 1 & 1 & 1 & 10 \end{pmatrix}$$

using the Householder algorithm. Verify your answer using the MATCOM program **HOUSEQRN** or Matlab program **qr** in the form $[Q, R] = \mathbf{qr}(A)$.