## MATH 326: Homework 8 <br> SPRING 2013

1. Consider the linear programming minimization problem

$$
\begin{array}{cc}
\min & c^{T} x \\
\text { s.t. } & A x \leq b \\
& x \geq 0
\end{array}
$$

Under what conditions should a non-basic variable enter the basis? State and prove an analogous theorem to Theorem we proved in class (see lecture from $3 / 4 / 2013$, the first theorem on the top of the page) using your observation. [Hint: Use the definition of reduced cost. Remember that it is $-\partial z / \partial x_{j}$. Remember, you are working with a minimization problem.]
2. We considered the following problem in HW \# 4 (see problems 2 and 3 ). You can use the results you obtained in that homework.

Assume that a leather company manufactures two types of belts: regular and deluxe. Each belt requires 1 square yard of leather. A regular belt requires 1 hour of skilled labor to produce, while a deluxe belt requires 2 hours of labor. The leather company receives 40 square yards of leather each week and a total of 60 hours of skilled labor is available. Each regular belt nets $\$ 3$ in profit, while each deluxe belt nets $\$ 5$ in profit. The company wishes to maximize profit.
(a) In problem 2(a) (HW \# 4), you were asked to ignore the divisibility issues and construct a linear programming problem whose solution will determine the number of each type of belt the company should produce. In part (b) of that problem, you converted the problem into the standard form.
(b) Use the simplex algorithm in tableau form to solve the problem. [Hint: you can let your starting feasible solution be the one in which all decision variables are zero and your slack variables are basic.]
(c) Draw the feasible region and the level curves of the objective function. Verify that the optimal solution you obtained through the simplex method is the point at which the level curves no longer intersect the feasible region in the direction following the gradient of the objective function.
3. Consider the problem

$$
\begin{array}{cc}
\min & z\left(x_{1}, x_{2}\right)=2 x_{1}-x_{2} \\
\text { s.t. } & x_{1}-x_{2}+s_{1}=1 \\
& 2 x_{1}+x_{2}-s_{2}=6 \\
& x_{1}, x_{2}, s_{1}, s_{2} \geq 0
\end{array}
$$

Using the entering variable rule you developed in Problem 1, show that the minimization problem has an unbounded feasible solution. Find an extreme direction for this set. [Hint: The minimum ratio test is the same for a minimization problem. Draw a picture and choose an initial extreme point as your starting basic feasible solution. Execute the simplex algorithm as we did in class and use Theorem about an extreme direction in an unbounded LP problem (see lecture from $3 / 22 / 2013$, the bottom of page 6) to find an extreme direction of the feasible region.]
4. In this exercise, we will modify the objective function from Problem 3. Consider the problem:

$$
\begin{array}{cc}
\min & z\left(x_{1}, x_{2}\right)=2 x_{1} \\
\text { s.t. } & x_{1}-x_{2}+s_{1}=1 \\
& 2 x_{1}+x_{2}-s_{2}=6 \\
& x_{1}, x_{2}, s_{1}, s_{2} \geq 0
\end{array}
$$

Construct a simplex tableau with basic feasible solution $x_{1}=0, x_{2}=6, s_{1}=7$ and $s_{2}=0$.
(a) Using this tableau, decide whether this problem has no solution, an unbounded solution, a unique solution or an infinite set of alternative optimal solutions.
(b) Whatever you decide for part (a), state what the solution is. Be sure to justify your answer using information from the tableau.

