Math 432 - Numerical Linear Algebra - Fall 2013

Homework 8

Assigned: Saturday, November 2, 2013

Due: **Friday, November 8, 2013**

1. Using the Matlab command **qr**, find the QR factorization of

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}.$$

For this matrix and with vector $b = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}^T$, using the results of QR factorization, find (i) orthonormal bases for $\mathcal{R}(A)$ and $\mathcal{N}(A^T)$, (ii) P_A and P_N, (iii) vectors b_R and b_N.

2. Suppose $A = QR$, where Q is $m \times n$ and R is $n \times n$. Show that if the columns of A are linearly independent, then R must be invertible. [*Hint*: Study the equation $Rx = 0$ and use the fact that $A = QR$.]

3. Let

$$x_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad x_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad x_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}.$$

The set $\{x_1, x_2, x_3\}$ is linearly independent and thus is a basis for a subspace W of $\mathbb{R}^4$. Use the Gram-Schmidt process to construct a set of mutually orthonormal vectors q_1, q_2, q_3, i.e. an orthonormal basis for W. Use your results to construct a QR factorization of the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

4. (*The purpose of this exercise is to compute the accuracy and efficiency of different methods for QR factorization of a matrix. MATCOM programs are available on the textbook's web site: http://www.siam.org/books/ot116/*)

   (a) Compute the QR factorization for each matrix A of the following data sets as follows:

      i. $[Q, R]$ = **qr**(A) from Matlab or $[Q, R]$ = **housqr**(A) or **housqrn** (MATCOM implementations of Householder's method).

      ii. $[Q, R]$ = **clgrsch**(A) (classical Gram-Schmidt implementation from MATCOM)

Method	$\|\|(\hat{Q})^T\hat{Q} - I\|\|_F$	$\frac{\|\|A - \hat{Q}\hat{R}\|\|_F}{\|\|A\|\|_F}$
housqr		
clgrsch		
mdgrsch		

Table 1: Comparison of different QR factorization methods.

iii. $[Q, R] =$ **mdgrsch**(A) from MATCOM (modified Gram-Schmidt implementation from MATCOM)

(b) Using the results of (a), complete the table above for each matrix. $\hat{Q}$ and $\hat{R}$ stand for the computed Q and R.

Data set:

i. $A = \text{rand}(25)$,

ii. A is a Hilbert matrix of order 25: $A =$ **hilb**(25) in Matlab,

iii. $A = \begin{pmatrix} 1 & 1 & 1 \\ 10^{-4} & 0 & 0 \\ 0 & 10^{-4} & 0 \\ 0 & 0 & 10^{-4} \end{pmatrix}$

iv. a Vandermonde matrix of order 25, that can be generated in Matlab as follows:

```
x=1:25;
A=fliplr(vander(x));
```