Math 571 - Functional Analysis I - Fall 2017

Homework 8
Due: **Friday, October 27, 2017**

1. (# 5, Section 2.8) Show that on any sequence space X we can define a linear functional f by setting $f(x) = \xi_n$ (n fixed), where $x = (\xi_j)$. Is f bounded if $X = l^\infty$?

2. (# 8, Section 2.8) The null space $N(M^*)$ of a set $M^* \subset X^*$ is defined to be the set of all $x \in X$ such that $f(x) = 0$ for all $f \in M^*$. Show that $N(M^*)$ is a vector space.

3. (# 3, Section 2.9) Let $\{f_1, f_2, f_3\}$ be the dual basis of $\{e_1, e_2, e_3\}$ for $\mathbb{R}^3$, where $e_1 = (1, 1, 1)$, $e_2 = (1, 1, -1)$, $e_3 = (1, -1, -1)$. Find $f_1(x)$, $f_2(x)$, $f_3(x)$, where $x = (1, 0, 0)$.

4. (# 7, Section 2.9) Find a basis for the null space of the functional f defined on $\mathbb{R}^3$ by $f(x) = \alpha_1\xi_1 + \alpha_2\xi_2 + \alpha_3\xi_3$, where $\alpha_1 \neq 0$ and $x = (\xi_1, \xi_2, \xi_3)$.

5. (# 11, Section 2.9) If x and y are different vectors in a finite dimensional vector space X, show that there is a linear functional f on X such that $f(x) \neq f(y)$.

6. (# 4, Section 2.10) Let X and Y be normed spaces and $T_n : X \to Y$ ($n = 1, 2, \ldots$) bounded linear operators. Show that convergence $T_n \to T$ implies that for every $\epsilon > 0$ there is an N such that for all $n > N$ and all x in any given closed ball we have $||T_n x - Tx|| < \epsilon$.

7. (# 8, Section 2.10) Show that the dual space of the space c_0 is l^1. (Cf. Prob. 1 in Section 2.3.)