MATH 326: HOMEWORK 9
SPRING 2013

1. Suppose I provide you with the following Tableau:

$$
\begin{bmatrix}
 & z & x_1 & x_2 & s_1 & s_2 & \text{RHS} \\
z & 1 & 0 & 0 & a & b & 2 \\
x_1 & 0 & 1 & 0 & -\frac{2}{3} & -\frac{1}{3} & 1 \\
x_2 & 0 & 0 & 1 & -\frac{1}{3} & -\frac{2}{3} & 1
\end{bmatrix}
$$

   (a) If I tell you this is an optimal tableau for a minimization problem, then what are the signs of $a$ and $b$?

   (b) Assuming I tell you that this tableau is optimal for some values of $a$ and $b$, what value of $a$ (or $b$) would allow you to decide that this problem has alternative optimal solutions?

2. Use the simplex algorithm to determine whether the following problem has alternative optimal solutions. If it does, determine two alternative optimal extreme point solutions.

$$
\begin{aligned}
\text{min} \quad & x_1 + 2x_2 \\
\text{s.t.} \quad & 4x_1 + 4x_2 \leq 6 \\
& x_1 \leq 4 \\
& x_2 \leq 2 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

3. Suppose that you have been asked to teach a section of Calculus I. You decide to give a combination of homework and quizzes to grade students. Each quiz will be worth 4 points, while each homework assignment will be worth 6 points. You wish to assign at least 100 points worth of graded work. Furthermore, you wish to assign at least 10 homework assignments. Each batch of quizzes takes 1 hour to grade, while each batch of homework assignments takes 2 hours to grade.

   (a) Construct a linear programming problem to determine the number of quizzes and homework assignments to give in order to minimize the amount of time you spend grading. **Note: For this problem, you may ignore integrality requirements and assume you can give half a quiz or homework assignment.**

   (b) Use the two-phase simplex algorithm to identify the optimal grading policy.

4. Suppose that I wish to start a new diet consisting of Raman noodles and salad. Each serving of Raman noodles costs \$1. Each serving of salad costs \$2. Suppose that I wish to consume at least 2 servings of salad per day (for vegetables). I also wish

to consume at least 1400 calories and 20 grams of protein. Each serving of Raman noodles contains 200 calories and contains 5 grams of protein. Each serving of salad contains 100 calories and 1 gram of protein. Assume I want to minimize the amount per day I spend on food.

(a) Construct a linear programming problem whose solution will provide the optimal diet for me.

(b) Use the two-phase simplex algorithm to identify the optimal diet. In your solution use the two "Row 0" method we discussed in class (lecture from 4/5/2013).