Math 432 - Numerical Linear Algebra - Fall 2013

<u>Homework 9</u>

Assigned: Saturday, November 9, 2013

Due: **Friday, November 15, 2013**

1. (a) Using the Matlab command **svd**, find the SVD of the following matrices:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, \quad A = (\,1 \quad 2 \quad 3\,)$$

$$A = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad A = \mathrm{diag}\,(\,1, 0, 2, 0, -5\,)$$

$$A = \begin{pmatrix} 1 & 1 \\ \epsilon & 0 \\ 0 & \epsilon \end{pmatrix}, \quad \epsilon = 10^{-5}.$$

   (b) Using the results of (a), find (i) rank, (ii) $||.||_2$ and $||.||_F$, (i) orthonormal bases for $\mathcal{R}(A)$ and $\mathcal{N}(A^T)$, (ii) P_A and P_N for each matrix.

2. Let A be an $m \times n$ matrix.

   Using the SVD of A, prove that

   (a) $||A^TA||_2 = ||A||_2^2$;

   (b) $\mathrm{Cond}_2(A^TA) = (\mathrm{Cond}_2(A))^2$;

   (c) $\mathrm{Cond}_2(A) = \mathrm{Cond}_2(U^TAV)$, where U and V are orthogonal.

3. (a) Let

$$A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix}$$

   Express A in terms of its singular values and singular vectors.

   (b) Compute $(A^TA)^{-1}$ using the SVD of A.

4. Let

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0.0001 & 1 & 1 \\ 0 & 0 & 0.0001 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   (a) Verify using the Matlab command **rank** that the $\mathrm{rank}(A) = 4$.

(b) Now run the following Matlab command: $[U, S, V] = \textbf{svd}(A)$. Set $S(4,4) = 0$; compute $B = U * S * V'$. What is the rank of B?

(c) What is the distance of B to A? What is the distance of A from the nearest singular matrix? What is that nearest singular matrix?

5. Compute the *rank* of each of the following matrices using the SVD of matrix A. Check your answers by using the Matlab command **rank** (which uses the singular values of A).

   Test data:

   (a) The Kahan matrix (see (7.9) pg. 217), with $n = 100$, and $c = 0.2$.

   (b) A 15×10 matrix A created as follows: $A = xy^T$, where

   $$x = \text{round}(10 * \text{rand}(15, 1)), \quad y = \text{round}(10 * \text{rand}(10, 1)).$$