Operator Identities

I $(D-a)\,\mathrm{e}^{ax}=0$

root a

II $(D-a)^n x^k\,\mathrm{e}^{ax}=0, \qquad k=0,1,2,\ldots,n-1$

roots $\underbrace{a,a,a,\ldots,a}_{n \text{ times}}$

III $(D^2+b^2)\begin{matrix}\cos bx\\ \sin bx\end{matrix}=0$

roots $\pm ib$

IV $(D^2+b^2)^n\begin{matrix}x^k\cos bx\\ x^k\sin bx\end{matrix}=0, \qquad k=0,1,2,\ldots,n-1$

roots $\underbrace{\pm ib,\pm ib,\ldots,\pm ib}_{n \text{ pairs}}$

V $[(D-a)^2+b^2]\begin{matrix}\mathrm{e}^{ax}\cos bx\\ \mathrm{e}^{ax}\sin bx\end{matrix}=0$

roots $a\pm ib$

VI $[(D-a)^2+b^2]^n\begin{matrix}x^k\,\mathrm{e}^{ax}\cos bx\\ x^k\,\mathrm{e}^{ax}\sin bx\end{matrix}=0, \qquad k=0,1,2,\ldots,n-1$

roots $\underbrace{a\pm ib,a\pm ib,\ldots,a\pm ib}_{n \text{ pairs}}$