Name: ______________________________

Math 310 - Differential Equations - Fall 2020

Matlab Project # 2 – due by September 22

1. Use Matlab and the function **eulerODE**, available on the course website, to solve problems # 17 and # 21 from Section 2.4 using Euler's method. Graph your numerical solutions using **plot** command. Put solutions with different h on the same plot using the command **hold on**. Discuss your results.

   For help with command **plot** or other functions, type **help plot** in the command line. An example on how to use **eulerODE** and other functions is available on the course web site - please see the program **main.m**. Use commands **legend**, **xlabel**, **ylabel**, **title** to mark solutions with different h, label your x and y axes and include a title, respectively.

2. Use Matlab and a function **impeuler** to solve problem # 26 from Section 2.5 using the modified (improved) Euler method. Graph your solution. Discuss your results.

3. Use Runge-Kutta 4th order method implemented in the function **rk** to solve problem # 29 from Section 2.6. Graphs and discuss your results.

Note. If you wish, you can write your own programs to implement Euler, modified Euler and Runge-Kutta 4th-order methods.

Example. Solve IVP $\frac{dy}{dx} = x + y$, $y(0) = 1$ numerically on $x \in [0, 5]$. The exact solution of this problem is $y = 2\,e^{x} + x - 1$. The results with $h = 1$ and $h = 0.1$ are shown below.

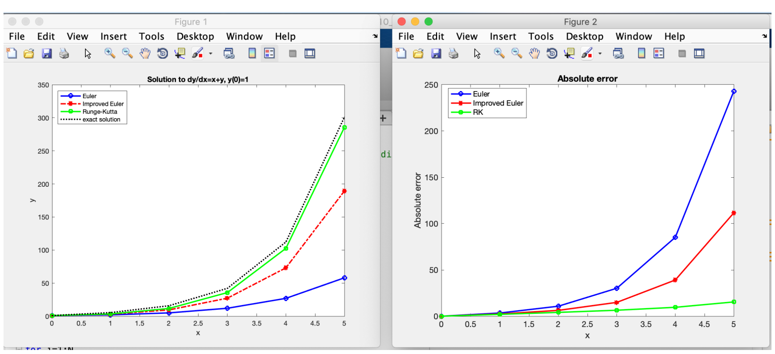

Figure 1: Solution $y(x)$ and the absolute error with $h = 1$

You can see in Fig. 1 that Euler's method deviates from the exact solution more than other methods and thus produces the largest error, as expected. Recall that the absolute error is defined as a difference between the exact solution and its approximation.

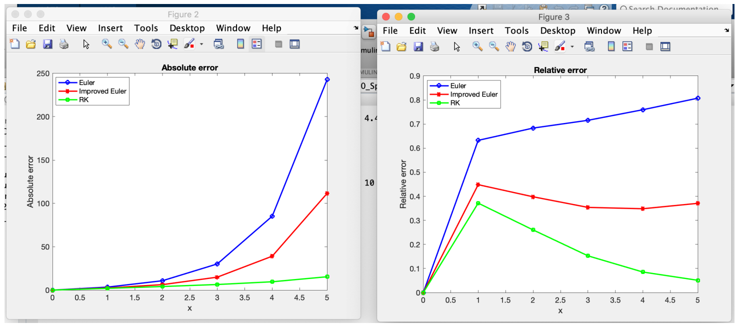

Figure 2: Absolute and relative errors with $h = 1$

The relative error is defined as a ratio of the absolute error and the exact solution. It is also the largest with Euler's method. See Fig. 2.

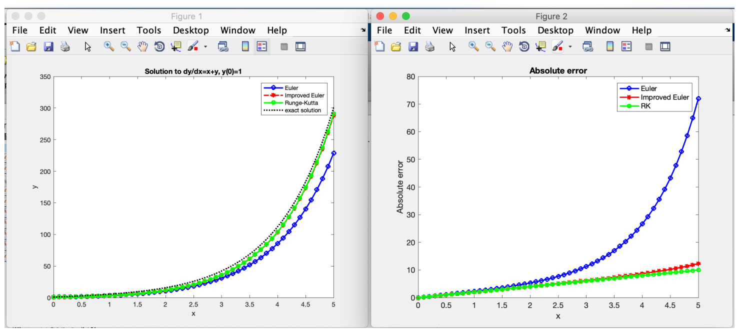

Figure 3: Solution $y(x)$ and the absolute error with $h = 0.1$

As the step size decreases to $h = 0.1$, the error overall decreases, but Euler's method is still the least accurate. You can see this in Figs 3 and 4 . Note that the solution using Runge-Kutta 4th order method is almost indistinguishable from the exact solution.

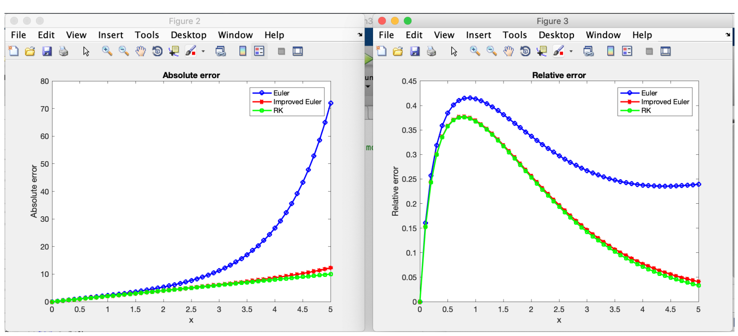

Figure 4: Absolute and relative errors with $h = 0.1$