Math 428/Eng 428/Phys 428 and Phys 528

Numerical Methods - Spring 2015

Practice Problems: Richardson Extrapolation

1. **(Richardson Extrapolation Applied to Differentiation).**

   (a) Suppose that $N(h)$ is an approximation to M for every $h > 0$ and that

   $$M = N(h) + K_1 h^1 + K_2 h^2 + K_3 h^3 + \dots$$

   for some constants K_1, K_2, K_3, ... Use the values $N(h)$, $N(\frac{h}{3})$, and $N(\frac{h}{9})$ to produce an $\mathcal{O}(h^3)$ approximation to M.

   (b) Recall that

   $$\frac{df(x_0)}{dx} = \frac{f(x_0+h) - f(x_0)}{h} + \sum_{i=2}^{\infty} \frac{h^{i-1}}{i!} f^{(i)}(x_0) \ .$$

   Use the formula you constructed in part (a) to construct an $O(h^3)$ approximation to $\frac{df(x_0)}{dx}$.

2. **(Richardson Extrapolation Applied to Solving IVPs).** Perform one step of Richardson's extrapolation to get an improved solution at $x = 1$, using values obtained with $h = 0.1$ and $h = 0.05$ by the second order Runge-Kutta method (Improved Euler method). Compare with exact solution.