#9 S7.4 $F(s) = \dfrac{1}{(s^2+9)^2} = \dfrac{1}{s^2+9} \cdot \dfrac{1}{s^2+9} = \mathcal{L}\{\frac{1}{3}\sin 3t\} \cdot \mathcal{L}\{\frac{1}{3}\sin 3t\}$

$\Rightarrow f(t) = \frac{1}{9}\sin 3t * \sin 3t = \frac{1}{9} \cdot \frac{1}{2}[\sin 3t - 3t \cdot \cos 3t] \cdot \frac{1}{3}$?

#13 S7.4 $F(s) = \dfrac{s}{(s-3)(s^2+1)} = \dfrac{1}{s-3} \cdot \dfrac{s}{s^2+1} = \mathcal{L}\{e^{3t}\} \cdot \mathcal{L}\{\cos \cancel{3}t\}$

$f(t) = e^{3t} * \cos t = \int_0^t \underbrace{e^{3(t-\tau)}}_{e^{3t}e^{-3\tau}} \cos\tau \, d\tau = e^{3t}\int_0^t e^{-3\tau}\cos\tau \, d\tau$

$f(t) * g(t) = \int_0^t f(t-\tau)\, g(\tau)\, d\tau = \int_0^t f(\tau)\, g(t-\tau)\, d\tau$

$\mathcal{L}\{t f(t)\} = -\frac{d}{ds}\overbrace{\mathcal{L}\{f(t)\}}^{F(s)}$

#17 S7.4 $f(t) = t e^{2t}\cos 3t$

$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$

$\mathcal{L}\{t \underbrace{e^{2t}\cos 3t}\} = -\frac{d}{ds}\mathcal{L}\{e^{2t}\cos 3t\} =$ $a=2$

$= -\frac{d}{ds}\left\{\dfrac{s-2}{(s-2)^2+3^2}\right\}$

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(\sigma)\,d\sigma$$

$$\lim_{t\to 0} \frac{f(t)}{t} \ \exists$$

#19 $f(t) = \dfrac{e^{3t}-1}{t}$

$$\mathcal{L}\left\{\frac{e^{3t}-1}{t}\right\} = \int_s^{\infty} \mathcal{L}\{e^{3t}-1\}\,ds =$$

$$\lim_{t\to 0} \frac{e^{3t}-1}{t} \underset{\text{rule}}{\overset{\text{l'Hop.}}{=}} \lim_{t\to 0} \frac{3e^{3t}}{1} = 3 \ \checkmark$$

$$= \int_s^{\infty} \left(\frac{1}{\sigma-3} - \frac{1}{\sigma}\right) d\sigma = \left(\ln(\sigma-3) - \ln\sigma\right)\Big|_{\sigma=s}^{\infty} = \ln\frac{\sigma-3}{\sigma}\Big|_{\sigma=s}^{\infty}$$

$$= \underbrace{\lim_{\sigma\to\infty} \underbrace{\ln\frac{\sigma-3}{\sigma}}_{\ln 1 = 0}}_{} - \ln\frac{s-3}{s} = -\ln\frac{s-3}{s}$$

$$= \ln \lim_{\sigma\to\infty} \frac{\sigma-3}{\sigma} = \ln 1 = 0$$

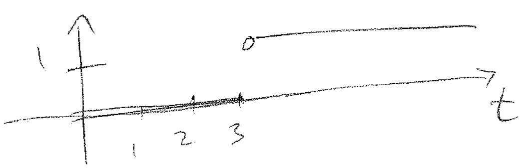
ln is cont fn

#11 S 7.5

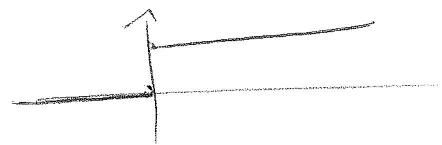
$$f(t) = \begin{cases} 2 & 0 \le t < 3 \\ 0 & t \geq 3 \end{cases}$$

$$F(s) = \mathcal{L}\{f(t)\} \stackrel{def}{=} \int_0^\infty f(t) e^{-3t}\,dt = \int_0^3 \underbrace{f(t)}_{= 2} e^{-3t}\,dt + \int_3^\infty f(t) e^{-3t}\,dt$$

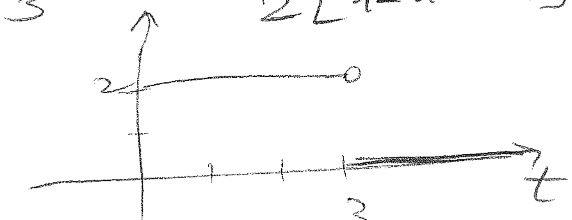
$u(t-3)$



$u(t)$

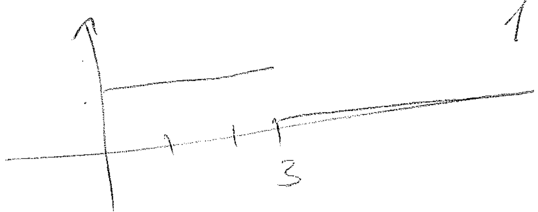


$2[1-u(t-3)]$

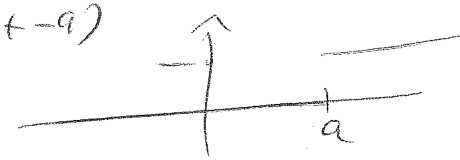


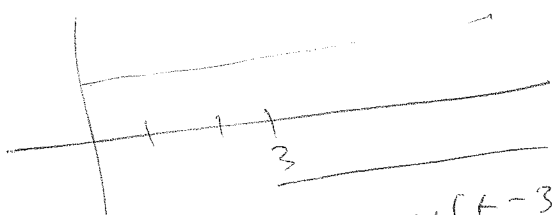
$2-2u(t-3)$

$1-u(t-3)$



$u(t-a)$





$-u(t-3)$

$$f(t) = 2[1-u(t-3)] = 2 - 2u(t-3)$$

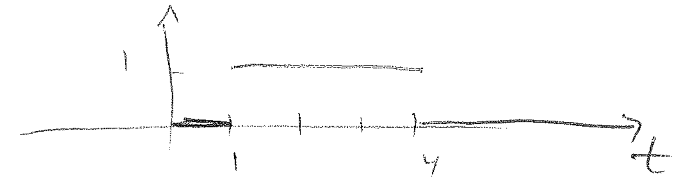
$$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$$

$$\mathcal{L}\{f(t)\} = \frac{2}{s} - \frac{2e^{-3s}}{s}$$

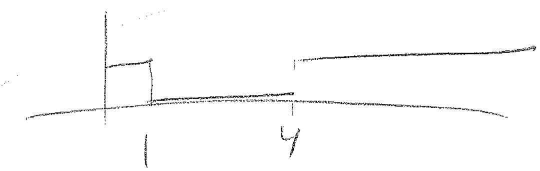
$$[\;] = (..)^+ - (..)^-$$

#12
S7.5

$$f(t) = \begin{cases} 0 & 0 \le t < 1 \\ 1, & 1 \le t \le 4 \\ 0, & t > 4 \end{cases}$$



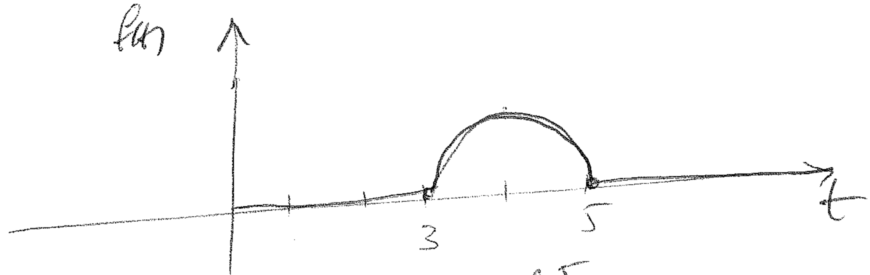
$f(t) = u(t-1) - u(t-4)$



$f(t) = 1 - u(t-1) + u(t-4)$

#18
S7.5

$$f(t) = \begin{cases} \cos\frac{\pi t}{2} & 3 \le t \le 5 \\ 0 & \text{otherwise} \end{cases}$$



$f(t) = [u(t-3) - u(t-5)] \cos\frac{\pi t}{2}$

By def, $\mathcal{L}\{f(t)\} = \int_3^5 e^{-st} \cos\frac{\pi t}{2}\, dt$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

cos

$\mathcal{L}\{u(t-3)\cos\frac{\pi t}{2}\}$

$\mathcal{L}\{u(t-a) f(t-a)\} = e^{-as} F(s)$

$$\cos\frac{\pi t}{2} = \cos\frac{\pi(t-3+3)}{2} = \cos\left[\frac{\pi(t-3)}{2} + \frac{3\pi}{2}\right] = \cos\frac{\pi(t-3)}{2} \cdot \cancel{\cos\frac{3\pi}{2}}^{\,0}$$

$$- \sin\frac{\pi(t-3)}{2} \cdot \overset{-1}{\sin\frac{3\pi}{2}} = -\sin\frac{\pi(t-3)}{2}$$

$$\mathcal{L}\{u(t-3)\cos\frac{\pi t}{2}\} = -\mathcal{L}\{u(t-3)\sin\frac{\pi(t-3)}{2}\} = -e^{-3s}\cdot\frac{\pi/2}{s^2+\left(\frac{\pi}{2}\right)^2}$$

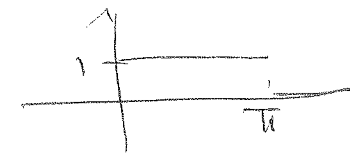
Similarly with ~~...~~ $u(t-5)\cos\frac{\pi t}{2} = -u(t-5)\sin\frac{\pi(t-5)}{2}$ ($\sin\frac{\pi t}{2}$)

#31 S7.5

$m\ddot{x} + cx' + kx = f(t)$, $x(0) = x'(0) = 0$

$m=1$, $k=4$, $c=0$

$$f(t) = \begin{cases} 1 & 0 \le t < \pi \\ 0 & t > \pi \end{cases}$$



$x'' + \cancel{4}4x = 1 - u(t-\pi)$ $\qquad f(t) = 1 - u(t-\pi)$

Let $X(s) = \mathcal{L}\{x(t)\}$

$$s^2X(s) - s\cancel{x(0)} - \cancel{x'(0)} + 4X(s) = \frac{1}{s} - \frac{e^{-\pi s}}{s}$$

$$(s^2+4)X(s) = \frac{1}{s} - \frac{e^{-\pi s}}{s}$$

$$X(s) = \frac{1}{s(s^2+4)} - e^{-\pi s}\frac{1}{s(s^2+4)}$$

$$\frac{1}{s(s^2+4)} = \frac{1}{s}\cdot\frac{1}{s^2+4}$$

$\mathcal{L}\{1\}$ $\quad \mathcal{L}\{\frac{1}{2}\sin 2t\}$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+4)}\right\} = 1 * \frac{1}{2}\sin 2t = \frac{1}{2}\int_0^t \sin 2\tau\, d\tau =$$

$$= -\frac{1}{\cancel{4}4}\cos 2\tau \Big|_{\tau=0}^{t} = -\frac{1}{4}(\cos 2t - 1)$$

(6

$$x(t) = \mathcal{L}^{-1}\{X(s)\} = -\frac{1}{4}(\cos 2t - 1) - \frac{1}{4}(\cos 2(t-\pi) - 1)\, u(t-\pi)$$

$$\mathcal{L}\{u(t-a) f(t-a)\} = e^{-as} F(s)$$