Aligned-Rank Example Calculations

Here we outline the steps to obtain aligned values from the example data, which are ranked and analyzed. We also point out how the corresponding parts of the SAS program are calculating the same quantities. Here are the data:

A/B	1	2	3	
1	1, 2, 3	4, 5, 6	7, 8, 9	$\widehat{\mu}_{1.} = 5$
2	10, 11, 12	13, 14, 15	20, 21, 22	$\widehat{\mu}_{2.} = 15.33$
	$\widehat{\mu}_{.1} = 6.5$	$\widehat{\mu}_{.2} = 9.5$	$\widehat{\mu}_{.3} = 14.5$	$\widehat{\mu}_{..} = \widehat{\mu} = 10.17$

The individual cell means are: $\widehat{\mu}_{11} = 2, \widehat{\mu}_{12} = 5, \widehat{\mu}_{13} = 8, \widehat{\mu}_{21} = 11, \widehat{\mu}_{22} = 14$, and $\widehat{\mu}_{23} = 21$. In the SAS program, the estimated cell means $\widehat{\mu}_{ij}$ are computed in the Proc Reg step shortly after the data are entered, and the estimated mean $\widehat{\mu}_{ij}$ is denoted by abij in the SAS program.

The row, column, and total means can be calculated from the cell means and are denoted by $\widehat{\mu}_{i.}, \widehat{\mu}_{.j}$, and $\widehat{\mu}_{.}$ respectively. These are calculated in the SAS program at the beginning of the Data ALPARS step and are called ai, bj, and mu, respectively. With this information we can calculate the $\widehat{\alpha}_i, \widehat{\beta}_j$, and $\widehat{\gamma}_{ij}$ values:

$$\widehat{\alpha}_1 = \widehat{\mu}_{1.} - \widehat{\mu} = 5 - 10.17 = -5.17, \;\; \widehat{\alpha}_2 = \widehat{\mu}_{2.} - \widehat{\mu} = 15.33 - 10.17 = 5.16$$

$$\widehat{\beta}_1 = \widehat{\mu}_{.1} - \widehat{\mu} = 6.5 - 10.17 = -3.67, \;\; \widehat{\beta}_2 = \widehat{\mu}_{.2} - \widehat{\mu} = 9.5 - 10.17 = -0.67,$$
$$\widehat{\beta}_3 = \widehat{\mu}_{.3} - \widehat{\mu} = 14.5 - 10.17 = 4.33$$

$$\begin{aligned}
\widehat{\gamma}_{11} &= \widehat{\mu}_{11} - \widehat{\mu}_{1.} - \widehat{\mu}_{.1} + \widehat{\mu} = 2 - 5 - 6.5 + 10.17 = .667, \\
\widehat{\gamma}_{12} &= \widehat{\mu}_{12} - \widehat{\mu}_{1.} - \widehat{\mu}_{.2} + \widehat{\mu} = 5 - 5 - 9.5 + 10.17 = .667, \\
\widehat{\gamma}_{13} &= \widehat{\mu}_{13} - \widehat{\mu}_{1.} - \widehat{\mu}_{.3} + \widehat{\mu} = 8 - 5 - 14.5 + 10.17 = -1.33, \\
\widehat{\gamma}_{21} &= \widehat{\mu}_{21} - \widehat{\mu}_{2.} - \widehat{\mu}_{.1} + \widehat{\mu} = 11 - 15.33 - 6.5 + 10.17 = -.667, \\
\widehat{\gamma}_{22} &= \widehat{\mu}_{22} - \widehat{\mu}_{2.} - \widehat{\mu}_{.2} + \widehat{\mu} = 14 - 15.33 - 9.5 + 10.17 = -.667, \\
\widehat{\gamma}_{23} &= \widehat{\mu}_{23} - \widehat{\mu}_{2.} - \widehat{\mu}_{.3} + \widehat{\mu} = 21 - 15.33 - 14.5 + 10.17 = 1.33.
\end{aligned}$$

These values are calculated in the SAS program in the Data ALPARS step and are called alphai, betaj, and gammaij.

Now that we have the parameter estimates, we can calculate the aligned values. This is done in the SAS program by merging the data set ALPARS that has the parameter estimates with the TABLE922 data set that has the data values, the merged data set is named BOTH. Recall that the formulas for the aligned values are:

$$\begin{aligned} A_{ijk} &= Y_{ijk} - \widehat{\mu} - \widehat{\beta}_j - \widehat{\gamma}_{ij}, \\ B_{ijk} &= Y_{ijk} - \widehat{\mu} - \widehat{\alpha}_i - \widehat{\gamma}_{ij}, \\ AB_{ijk} &= Y_{ijk} - \widehat{\mu} - \widehat{\alpha}_i - \widehat{\beta}_j. \end{aligned}$$

To calculate these in SAS, we have to include indicator variables so that only the appropriate $\widehat{\alpha}_i$, $\widehat{\beta}_j$, or $\widehat{\gamma}_{ij}$ terms are used in calculating an aligned value. Here is an example of calculating an A_{ijk} term this way:

$$\begin{aligned} A_{123} = Y_{123} &- \widehat{\mu} - \widehat{\beta}_1 I(B\ level\ 1) - \widehat{\beta}_2 I(B\ level\ 2) - \widehat{\beta}_3 I(B\ level\ 3) \\ &- \widehat{\gamma}_{11} I(A\ level\ 1) I(B\ level\ 1) - \widehat{\gamma}_{12} I(A\ level\ 1) I(B\ level\ 2) \\ &- \widehat{\gamma}_{13} I(A\ level\ 1) I(B\ level\ 3) - \widehat{\gamma}_{21} I(A\ level\ 2) I(B\ level\ 1) \\ &- \widehat{\gamma}_{22} I(A\ level\ 2) I(B\ level\ 2) - \widehat{\gamma}_{23} I(A\ level\ 2) I(B\ level\ 3). \end{aligned}$$

Since A_{123} is in level 1 of factor A and level 2 of factor B, we have $I(A\ level\ 1) = I(B\ level\ 2) = 1$, so most of the terms in the expression above become 0, leaving

$$\begin{aligned} A_{123} &= Y_{123} - \widehat{\mu} - \widehat{\beta}_2 - \widehat{\gamma}_{12} \\ &= 6 - 10.17 - (-0.67) - (.67) = -4.17. \end{aligned}$$

This approach is used to calculate all of the A_{ijk}, B_{ijk}, and AB_{ijk} values in the SAS program in the data set BOTH, they are denoted by the names alignA, alignB, and alignAB, respectively. These aligned values are then ranked, and the aligned ranks are analyzed in Proc GLM. Remember that although we conduct three separate factorial analyses, in each analysis we are only interested in one of the three F tests.