1 The Percentile, Residual, and BCA methods for Bootstrap confidence intervals

1.1 The Percentile method

The Percentile method is probably the most intuitively clear method for creating a bootstrap confidence interval. It involves creating many ($\geq 5000$) bootstrap samples, and calculating a $\hat{\theta}_b$ value for each bootstrap sample. Then a 95% interval for $\theta$, for example, would be:

$$\hat{\theta}_{b,0.025} < \theta \leq \hat{\theta}_{b,0.975}$$

1.2 The Residual method

The Residual method is somewhat similar to the Percentile method, except it is based on a bootstrap distribution of residuals from the original estimate $\hat{\theta}$. For this method, many ($\geq 5000$) bootstrap samples are created and $e_b = \hat{\theta}_b - \hat{\theta}$ is calculated for each bootstrap sample. A 95% interval for $\theta$, for example, would then be:

$$\hat{\theta} - e_{b,0.975} \leq \theta < \hat{\theta} - e_{b,0.025}$$

1.3 The BCA method

The BCA method creates an interval similar to the percentile interval, except that instead of having the interval be $\hat{\theta}_{b,0.025} < \theta \leq \hat{\theta}_{b,0.975}$, it is like $\hat{\theta}_{b,\alpha_L} < \theta \leq \hat{\theta}_{b,\alpha_U}$, where $L$ and $U$ are chosen to make the interval median unbiased and adjusted for skewness. The values $\alpha_L$ and $\alpha_U$ are chosen to have the same cumulative probability as $z_L$ and $z_U$, defined as:

$$z_L = \frac{z_0 - z_{1-\alpha/2}}{1 - a(z_0 - z_{1-\alpha/2})} + z_0 \quad \text{and} \quad z_U = \frac{z_0 + z_{1-\alpha/2}}{1 - a(z_0 + z_{1-\alpha/2})} + z_0.$$  

The value $z_0$ measures median unbiasedness and is defined via: $P(Z \leq z_0) = p_0$, where $p_0$ is the proportion of $\hat{\theta}_b \leq \hat{\theta}$. The value $a$ measures skewness of the data and is given by:
\[ a = \frac{\sum (\hat{\theta}_(-i) - \hat{\theta}_(-i))^3}{6 \left[ \sum (\hat{\theta}_(-i) - \hat{\theta}_(-i))^2 \right]^{3/2}}, \]

where \( \hat{\theta}_{-i} \) is the estimate of \( \theta \) computed without the \( i^{th} \) observation, \( X_i \), and \( \hat{\theta}_(-i) \) is the mean of the \( \hat{\theta}_{-i} \) values. Note that if \( a = 0 \) and \( z_0 = 0 \), then the BCA method is the same as the percentile method.