Fisher's Exact Test; Tests for Tables with Ordered Categories

Fisher's Exact Test

Recall that for the permutation tests for contingency tables that we just studied, we permute the data while keeping the row and column marginal totals fixed. When we have the special case of a table of size 2x2, the table has only 1 degree of freedom, and we can use probability calculations from the hypergeometric distribution to conduct an exact test more easily. **Fisher's exact test** is the name of the permutation test applied to a 2x2 table.

A general 2x2 table has the following form:

Row/Column	1	2	Totals
1	x	$n - x$	n
2	$M - x$	$x - [M + n - N]$	$N - n$
Totals	M	$N - M$	N

,

where x satisfies $\max(0, n - N + M) \leq x \leq \min(n, M)$. The probability of observing a table with a given value of x can be calculated using the hypergeometric distribution as:

$$P(x|n, M, N) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}.$$

For example, our football data is:

Conference/Margin	Margin ≤ 7	Margin > 7	
AFC	5	0	5
NFC	2	4	6
	7	4	11

,

so that $x = 5$, $n = 5$, $M = 7$, and $N = 11$. Thus the probability of the observed table is:

$$P(x = 5|n = 5, M = 7, N = 11) = \frac{\binom{7}{5}\binom{4}{0}}{\binom{11}{5}} =$$

$$= \frac{\frac{7!}{5!2!}\frac{4!}{0!4!}}{\frac{11!}{5!6!}} = \frac{\frac{6\cdot 7}{2}\frac{4!}{4!}}{\frac{11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6!}{5\cdot 4\cdot 3\cdot 2\cdot 6!}} = \frac{3 \cdot 7}{11 \cdot 3 \cdot 2 \cdot 7} = \frac{1}{22} \approx .045.$$

The possible values of x are $\max(0, 5 - 11 + 7) \leq x \leq \min(5, 7)$ or $1 \leq x \leq 5$. We can calculate table probabilities and other quantities of interest for the football data to get:

x	table χ^2	table prob.	Left-sided prob.	Right-sided prob.	Two-sided prob.
5	5.23	.045	1.00	.045	.061
4	1.06	.303	.954	.348	.546
3	.052	.455	.652	.803	1.00
2	2.21	.182	.197	.985	.242
1	7.54	.015	.015	1.00	.015

Note that the left-sided probability is $P(X \leq x_{obs})$, the right-sided probability is $P(X \geq x_{obs})$, and the two-sided probability is $P(\chi^2 \geq \chi^2_{obs})$. Depending upon the question of interest, we may use any of these three probabilities to calculate our P value. For the usual permutation chi-squared test from the last section, the only P value that we can calculate is the two-sided probability.

Tables with Ordered Categories

Tables can be either singly-ordered or doubly ordered. For these situations we can use tests that we have already seen, the Kruskal-Wallis test for singly-ordered tables and the Jonckheere-Terpstra test for doubly-ordered tables. In either situation we use the test for data with ties.