Page's test

We also have a test for the randomized complete block design with ordered alternatives. If our model is:

$$X_{ij} = \mu + t_i + b_j + \varepsilon_{ij},$$

and we have an ordered alternative such as

$$H_a : t_1 \leq t_2 \leq \cdots \leq t_k \text{ (or the reverse)},$$

then we can use Page's statistic:

$$PG = \sum_{i=1}^{k} i R_i,$$

where R_i is the sum of the ranks in the ith group. As with many of the other tests we have studied, we can either use a permutation approach to obtain an exact P value (or an estimate of the exact P value), or we can use a large-sample approximate Z test. For the large sample approximation we use:

$$E(PG) = \frac{bk(k+1)^2}{4} \text{ and } var(PG) = \frac{(k-1)k(k+1)}{12} \sum_{j=1}^{b} S_{B_j}^2,$$

where $S_{B_j}^2$ is the sample variance of the ranks or adjusted ranks in the jth block.