Paired-Comparison Permutation test

Paired comparison and block designs are important designs for controlling variation in an experiment. For paired-comparison designs, we can summarize the data in terms of differences D_i and their sample mean $\overline{D}$, as shown in Table 4.1.1 in the text. Given these differences, under the null hypothesis of no difference in treatments, the occurence of $-D_i$ is just as likely as observing D_i. Thus the permutation distribution used to test the null hypothesis consists of all 2^n possible arrangements of $+$ and $-$ signs attached to the $|D_i|$ values, as outlined on pages 110-111. Alternative versions of the test statistic are S_+ and S_-, the sum of the positive and negative differences, respectively.

The null hypothesis

If $F(x)$ is the cdf of the population of differences D from which our data was randomly sampled, the null hypothesis is that $F(x)$ is symmetric about zero. This can also be expressed as:

$$H_0 : F(x) = 1 - F(-x)$$

As we have seen previously, upper-tail, lower-tail, and two-tail alternatives are all used. Also, there is the special case of the shift alternative where $F(x) = G(x - \Delta)$ as discussed in the text.

Random sampling from the permutation distribution of D_i

The text discusses how to randomly sample either $\overline{D}$ or S_+ values from the permutation distribution under H_0. For example, S_+ can be sampled via:

$$S_+ = \sum_{i=1}^{n} V_i |D_i|,$$

where the V_i are independent random variables such that $V_i = 0$ or $V_i = 1$ each having probability .5 .

Large-Sample Approximations

These are given in the text, for example using the following expressions lead to a Z statistic for S_+:

$$E(S_+) = \frac{1}{2} \sum_{i=1}^{n} |D_i| \text{ and } var(S_+) = \frac{1}{4} \sum_{i=1}^{n} |D_i|^2$$

The text also discusses the use of the paired-data procedure for testing the median of a symmetric population.