Methods for Randomized Complete Block Designs

Randomized complete block (RCB) designs reduce error variation by using blocks that are homogeneous, have the same number of experimental units as treatment levels, and by randomizing the treatments to the units within the blocks. The standard notation for an RCB is illustrated in the text. The model for an RCB design is:

$$X_{ij} = \mu + t_i + b_j + \varepsilon_{ij},$$

where the terms are defined as usual, in particular, ε_{ij} are i.i.d. random variables with median 0. If the errors are normally distributed with mean 0 and common variance σ^2, then we test the null hypothesis $H_0 : t_1 = t_2 = ... = t_k$ versus H_a : Not all t_i are equal, with the F statistic:

$$F = \frac{b \sum_{i=1}^{k} (\overline{X}_{i.} - \overline{X})^2/(k-1)}{\sum_{i=1}^{k} \sum_{j=1}^{b} (X_{ij} - \overline{X}_{i.} - \overline{X}_{.j} + \overline{X})^2/[(k-1)(b-1)]},$$

which has an F distribution with $k-1$ and $(k-1)(b-1)$ degrees of freedom under H_0.

A Permutation RCB F test

The key to the permutation RCB F test is to have the permutation method mimic the original randomization, which is within blocks. Thus, permuted data sets are obtained by permuting the observations separately within each block. With k treatments, there are $k!$ permutations possible in each block, and since there are b blocks, there are a total of $(k!)^b$ elements in the permutation distribution for a given data set. The procedure is similar to permutation tests we have done previously, and is outlined in the text. Note that alternate statistics such as $SST^* = \sum_{i=1}^{k} (\overline{X}_{i.} - \overline{X})^2$ or $SSX^* = \sum_{i=1}^{k} (\overline{X}_{i.})^2$ may be used for the test. Generally a random sample from the permutation distribution will be obtained instead of completely enumerating the distribution. The text also covers a permutation HSD procedure to use with the RCB, based on the $Q^* = \max_{ij} \left| \overline{X}_{i.} - \overline{X}_{j.} \right|$ statistic.

Friedman's test for RCB designs

Friedman's test is a rank-based test for an RCB design, which proceeds by ranking the values separately within each block. In the case where there are no ties within blocks, the Friedman statistic is:

$$FM = \frac{12b}{k(k+1)} \sum_{i=1}^{k} \left(\overline{R}_i - \frac{k+1}{2} \right)^2 .$$

A permutation p-value can be obtained for the Friedman test, or a large-sample approximate p-value can be calculated using a chi-squared distribution with $k-1$ degrees of freedom. Note that the main part of the FM statistic is essentially the numerator of the RCB F statistic based on ranks. When there are ties within blocks, adjusted ranks are then used and the Friedman statistic then is:

$$FM_{ties} = \frac{b^2}{\sum\limits_{j=1}^{b} S_{Bj}^2} \sum_{i=1}^{k} \left(\overline{R}_i - \frac{k+1}{2} \right)^2 ,$$

where S_{Bj}^2 is the sample variance of the adjusted ranks within block j.