The Signed-Rank test

As we have seen before, the permutation test for raw-data paired-comparisons can be unduly affected by outliers, so a rank-based test will be useful. This leads us to the Wilcoxon signed-rank test. For this test, we rank the absolute value of the differences, and then add the sign of the difference to its' rank. The Wilcoxon signed-rank statistic SR_+ is the sum of the positive signed ranks. We can obtain permutation p values for this test by either enumerating the permutation distribution of all 2^n possible $+/-$ combinations of ranks R_i, or we can obtain a sample from this distribution, as outlined in the text. When there are no ties a large sample approximation may be used with:

$$E(SR_+) = \frac{n(n+1)}{4}, \; Var(SR_+) = \frac{n(n+1)(2n+1)}{24},$$

and

$$Z = \frac{SR_+ - E(SR_+)}{\sqrt{Var(SR_+)}}.$$

The Signed-Rank test with ties

There are two types of ties that can occur. For D_i values that are equal, but not 0, the usual adjusted-ranks can be used. However, if the raw data are tied so that some D_i values are 0, then there are two approaches: i) ranking with zeros, or ii) ranking without zeros. These methods are illustrated in the text. Again, either permutation p values can be obtained or a large-sample approximation can be used. Note that for the large-sample approximation,

$$E(SR_+) = \frac{1}{2}\sum_{i=1}^{m} |\text{Signed Ranks}| \text{ and } Var(SR_+) = \frac{1}{4}\sum_{i=1}^{m} (\text{Signed Ranks})^2,$$

where m is the number of nonzero differences.

The Sign Test

The sign test counts the number of positive differences, denoted by SN_+. Under the null hypothesis of no difference, SN_+ follows a binomial distribution with $p = .5$. Thus p values can be computed directly,

$$P(SN_+ \geq k) = \sum_{i=k}^{n} \binom{n}{i} (.5)^n$$

or via a normal approximation using,

$$Z = \frac{SN_+ - n/2}{\sqrt{n/4}}.$$

General Scoring Systems, selecting a Paired-Comparison test